The thermodynamic cuboctahedron is a three-dimensional generalization of the thermodynamic square which is a mnemonic device for generating thermodynamic equations, particularly the Maxwell relations. Specifications for the construction of a thermodynamic cuboctahedron are given in this paper.

In 1929, Professor Max Born delivered a lecture on Maxwell's relations in which he used a diagram like the one in Figure 1. F. O. Koenig has elaborated on Born's idea and has provided geometrizations of other thermodynamic relationships. Koenig has even used a thermodynamic octahedron as a geometrical element but in a way very different from the way the thermodynamic cuboctahedron is used in this paper.

The "thermodynamic square" contains a great deal of information:

1) The four fundamental thermodynamical potentials, $U$, $A$, $H$, and $G$ are exhibited at the edges of the square and are flanked at the corners by the thermodynamic parameters upon which they depend

$$U = U(S, V)$$
$$A = A(T, V)$$
$$H = H(S, P)$$
$$G = G(T, P)$$

2) The two arrows provide plus and minus signs for the differential form of the first law. If an arrow points away from a thermodynamic parameter, then the differential of that parameter has a $+$ sign. If the arrow points toward a thermodynamic parameter, then the differential of that parameter has a $-$ sign. This convention gives

$$dU = TdS - PdV$$
$$dA = SdT - PdV$$
$$dH = TdS + VdP$$
$$dG = SdT + VdP$$

3) The Maxwell relations can be obtained. They depend only upon the parameters at the corners of the square, and the orientation of the arrows. Figure 2 depicts two situations which lead to the Maxwell's relations

$$\left( \frac{\partial V}{\partial S} \right)_P = \left( \frac{\partial T}{\partial P} \right)_S$$
$$\left( \frac{\partial S}{\partial P} \right)_T = - \left( \frac{\partial V}{\partial T} \right)_P$$

The first relation has the same sign on both sides of the equation because the arrows in Figure 2 are oriented symmetrically. The second relation has opposite signs on the two sides of the equation because the arrows are oriented asymmetrically in the corresponding diagram of Figure 2.

In order to include the mole number, $N$, the chemical potential, $\mu$, and the grand canonical potential, $\Omega = \Omega(T, V, \mu)$, additional thermodynamic squares are introduced as is depicted in Figure 3.

From the thermodynamic squares in Figure 3, relevant equations for the grand canonical potential are obtained. Using both squares in Figure 3 and the same sign convention as was used in Figure 1, gives

$$d\Omega = SdT - PdV - Nd\mu$$

Note that one square in Figure 3 gives $d\Omega = -SdT - Nd\mu$ while the other square gives $d\Omega = -PdV - Nd\mu$. Consequently, in order to get the full, three parameter dependence for $\Omega$ or any of the other thermodynamic potentials, two squares must be used simultaneously. The squares in Figure 3 also provide more Maxwell relations

$$\left( \frac{\partial N}{\partial S} \right)_\mu = - \left( \frac{\partial T}{\partial \mu} \right)_S$$
$$\left( \frac{\partial S}{\partial \mu} \right)_T = \left( \frac{\partial N}{\partial T} \right)_\mu$$
$$\left( \frac{\partial V}{\partial N} \right)_P = \left( \frac{\partial \mu}{\partial P} \right)_N$$
$$\left( \frac{\partial N}{\partial P} \right)_\mu = - \left( \frac{\partial V}{\partial \mu} \right)_P$$
The additional thermodynamic potentials, $U', U'', U'''$, which appear in Figure 3 are defined by the Legendre transformations of $U$ given by

\[
\begin{align*}
U' &= U - N\mu \\
U'' &= U + PV - N\mu \\
U''' &= U + TS + PV - N\mu = 0
\end{align*}
\]

The first law in differential form, $dU = TdS - PdV + \mu dN$, and $U'''' = 0$ because of the Euler relation for first-order homogeneous functions which implies that:

\[
U'''' = U - TS + PV - N\mu = 0.
\]