CHAPTER 1

MEASUREMENT

Despite the mathematical beauty of some of its most complex and abstract theories, including those of elementary particles and general relativity, physics is above all an experimental science. It is therefore critical that those who make precise measurements be able to agree on standards in which to express the results of those measurements, so that they can be communicated from one laboratory to another and verified.

In this chapter we begin our study of physics by introducing some of the basic units of physical quantities and the standards that have been accepted for their measurement. We consider the proper way to express the results of calculations and measurements, including the appropriate dimensions and number of significant figures. We discuss and illustrate the importance of paying attention to the dimensions of the quantities that appear in our equations. Later in the text, other basic units and many derived units are introduced as they are needed.

1-1 THE PHYSICAL QUANTITIES, STANDARDS, AND UNITS

The building blocks of physics are the quantities that we use to express the laws of physics. Among these are length, mass, time, force, speed, density, resistivity, temperature, luminous intensity, magnetic field strength, and many more. Many of these words, such as length and force, are part of our everyday vocabulary. You might say, for example: “I will go to any length to help you as long as you do not force me to do so.” In physics, however, we must not be misled by the everyday meanings of these words. The precise scientific definitions of length and force have no connection at all with the uses of these words in the quoted sentence.

We can define an algebraic quantity, for instance, $L$ for length, any way we choose, and we can assume it is exactly known. However, when we try to assign a unit to a particular value of that quantity, we run into the difficulty of establishing a standard, so that those who have need of comparing one length with another will agree on the units of measurement. At one time, the basic unit of length was the yard, determined by the size of the king’s waistline. You can easily see the problems with such a standard: it is hardly accessible to those who need to calibrate their own secondary standards, and it is not invariable to change with the passage of time.

Fortunately, it is not necessary to define and agree on standards for every physical quantity. Some elementary quantities may be easier to establish as standards, and more complex quantities can often be expressed in terms of the elementary units. Length and time, for example, were for many years among the most precisely measurable physical quantities and were generally accepted as standards. Speed, on the other hand, was less precisely measurable and therefore was treated as a derived unit (speed = length/time). Today, however, measurements of the speed of light have reached a precision beyond that of the former standard of length; we still treat length as a fundamental unit, but the standard for its measurement is now derived from the standards of speed and time.

The basic problem is therefore to choose the smallest possible number of physical quantities as fundamental and to agree on standards for their measurement. These standards should be both accessible and invariable, which may be difficult to satisfy simultaneously. If the standard kilogram, for instance, is to be an invariable object, it must be inaccessible and must be kept isolated beyond the effects of handling and corrosion.

Agreement on standards has been accomplished through a series of international meetings of the General Conference on Weights and Measures beginning in 1889; the 19th meeting was held in 1991. Once a standard has been accepted, such as the second as a unit of time, then we can apply the unit to a vast range of measurements.
from the lifetime of the proton (greater than $10^{40}$ seconds) to the lifetime of the least stable particles that can be produced in our laboratories (about $10^{-23}$ second). When we express such a value as $10^{40}$ in units of seconds, what we mean is that the ratio between the lifetime of the proton and the time interval that is arbitrarily defined as the standard second is $10^{40}$. To accomplish such a measurement, we must have a way of comparing laboratory measuring instruments with the standard. Many of these comparisons are indirect, for no single measuring instrument is capable of operating precisely over 40 orders of magnitude. Nevertheless, it is essential to the progress of science that, when a researcher records a particular time interval with a laboratory instrument, the reading can in some way be connected to a calibration based on the standard second.

The quest for more precise or accessible standards is itself an important scientific pursuit, involving physicists and other researchers in laboratories throughout the world. In the United States, laboratories of the National Institute of Standards and Technology (formerly the National Bureau of Standards) are devoted to maintaining, developing, and testing standards for basic researchers as well as for scientists and engineers in industry. Improvements in our standards in recent years have been dramatic: since the first edition of this textbook (1960), the precision of the standard second has improved by more than a factor of 1000.

### 1-2 THE INTERNATIONAL SYSTEM OF UNITS*

The General Conference on Weights and Measures, at meetings during the period 1954–1971, selected as base units the seven quantities displayed in Table 1. This is the basis of the International System of Units, abbreviated SI from the French *Le Système International d’Unités.*

Throughout the book we give many examples of SI derived units, such as speed, force, and electric resistance, that follow from Table 1. For example, the SI unit of force, called the newton (abbreviation N), is defined in terms of the SI base units as

$$1 \text{ N} = 1 \text{ kg} \cdot \text{m/s}^2$$

as we shall make clear in Chapter 5.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>SI Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td>second $s$</td>
</tr>
<tr>
<td>Length</td>
<td>meter $m$</td>
</tr>
<tr>
<td>Mass</td>
<td>kilogram $kg$</td>
</tr>
<tr>
<td>Amount of substance</td>
<td>mole $mol$</td>
</tr>
<tr>
<td>Thermodynamic temperature</td>
<td>kelvin $K$</td>
</tr>
<tr>
<td>Electric current</td>
<td>ampere $A$</td>
</tr>
<tr>
<td>Luminous intensity</td>
<td>candela $cd$</td>
</tr>
</tbody>
</table>


If we express physical properties such as the output of a power plant or the time interval between two nuclear events in SI units, we often find very large or very small numbers. For convenience, the General Conference on Weights and Measures, at meetings during the period 1960–1975, recommended the prefixes shown in Table 2. Thus we can write the output of a typical electrical power plant, $1.3 \times 10^9$ watts, as 1.3 gigawatts or 1.3 GW. Similarly, we can write a time interval of the size often encountered in nuclear physics, $2.35 \times 10^{-9}$ seconds, as 2.35 nanoseconds or 2.35 ns. Prefixes for factors greater than unity have Greek roots, and those for factors less than unity have Latin roots (except femto and atto, which have Danish roots).

To fortify Table 1 we need seven sets of operational procedures that tell us how to produce the seven SI base units in the laboratory. We explore those for time, length, and mass in the next three sections.

Two other major systems of units compete with the International System (SI). One is the Gaussian system, in terms of which much of the literature of physics is expressed. We do not use this system in this book. Appendix G gives conversion factors to SI units.

The second is the British system, still in daily use in the United States. The basic units, in mechanics, are length (the foot), force (the pound), and time (the second). Again Appendix G gives conversion factors to SI units. We use SI units in this book, but we sometimes give the British equivalents, to help those who are unaccustomed to SI units to acquire more familiarity with them. In only three countries [Myanmar (Burma), Liberia, and the United States] is a system other than SI used as the accepted national standard of measurement.

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**Sample Problem 1** Any physical quantity can be multiplied by 1 without changing its value. For example, $1 \text{ min} = 60 \text{ s}$, so $1 = 60 \text{ s/}1 \text{ min}$; similarly, $1 \text{ ft} = 12 \text{ in}$, so $1 = 1 \text{ ft/}12 \text{ in}$. Using appropriate conversion factors, find (a) the speed in meters per second equivalent to 55 miles per hour, and (b) the volume in cubic centimeters of a tank that holds 16 gallons of gasoline.
TABLE 2 SI PREFIXES

<table>
<thead>
<tr>
<th>Factor</th>
<th>Prefix</th>
<th>Symbol</th>
<th>Factor</th>
<th>Prefix</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{18}$</td>
<td>exa-</td>
<td>E</td>
<td>$10^{-1}$</td>
<td>deci-</td>
<td>d</td>
</tr>
<tr>
<td>$10^{15}$</td>
<td>peta-</td>
<td>P</td>
<td>$10^{-2}$</td>
<td>centi-</td>
<td>c</td>
</tr>
<tr>
<td>$10^{12}$</td>
<td>tera-</td>
<td>T</td>
<td>$10^{-3}$</td>
<td>milli-</td>
<td>m</td>
</tr>
<tr>
<td>$10^{9}$</td>
<td>giga-</td>
<td>G</td>
<td>$10^{-6}$</td>
<td>micro-</td>
<td>μ</td>
</tr>
<tr>
<td>$10^{6}$</td>
<td>mega-</td>
<td>M</td>
<td>$10^{-9}$</td>
<td>nano-</td>
<td>n</td>
</tr>
<tr>
<td>$10^{3}$</td>
<td>kilo-</td>
<td>k</td>
<td>$10^{-12}$</td>
<td>pico-</td>
<td>p</td>
</tr>
<tr>
<td>$10^{2}$</td>
<td>hecto-</td>
<td>h</td>
<td>$10^{-15}$</td>
<td>femto-</td>
<td>f</td>
</tr>
<tr>
<td>$10^{1}$</td>
<td>deka-</td>
<td>da</td>
<td>$10^{-18}$</td>
<td>atto-</td>
<td>a</td>
</tr>
</tbody>
</table>

* In all cases, the first syllable is accented, as in na’-no-me’-ter. Prefixes commonly used in this book are shown in boldfaced type.

Solution  
(a) For our conversion factors, we need (see Appendix G) 1 mi = 1609 m (so that 1 = 1609 m/1 mi) and 1 h = 3600 s (so 1 = 1 h/3600 s). Thus

\[
\text{speed} = 55 \frac{\text{mi}}{\text{h}} \times \frac{1609 \text{ m}}{1 \text{ mi}} \times \frac{1 \text{ h}}{3600 \text{ s}} = 25 \text{ m/s}. \]

(b) One fluid gallon is 231 cubic inches, and 1 in. = 2.54 cm. Thus

\[
\text{volume} = 16 \text{ gal} \times \frac{231 \text{ in.}^3}{1 \text{ gal}} \times \left( \frac{2.54 \text{ cm}}{1 \text{ in.}} \right)^3 = 6.1 \times 10^4 \text{ cm}^3. \]

Note in these two calculations how the unit conversion factors are inserted so that the unwanted units appear in one numerator and one denominator, and thus cancel.

TABLE 3 SOME MEASURED TIME INTERVALS

<table>
<thead>
<tr>
<th>Time Interval</th>
<th>Seconds</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lifetime of proton</td>
<td>$&gt;10^{40}$</td>
</tr>
<tr>
<td>Half-life of double beta decay of $^{82}\text{Se}$</td>
<td>$3 \times 10^{27}$</td>
</tr>
<tr>
<td>Age of universe</td>
<td>$5 \times 10^{17}$</td>
</tr>
<tr>
<td>Age of pyramid of Cheops</td>
<td>$1 \times 10^{11}$</td>
</tr>
<tr>
<td>Human life expectancy (U.S.A.)</td>
<td>$2 \times 10^9$</td>
</tr>
<tr>
<td>Time of Earth’s orbit around the Sun (1 year)</td>
<td>$3 \times 10^7$</td>
</tr>
<tr>
<td>Time of Earth’s rotation about its axis (1 day)</td>
<td>$9 \times 10^4$</td>
</tr>
<tr>
<td>Period of typical low-orbit Earth satellite</td>
<td>$5 \times 10^3$</td>
</tr>
<tr>
<td>Time between normal heartbeats</td>
<td>$8 \times 10^{-1}$</td>
</tr>
<tr>
<td>Period of concert-A tuning fork</td>
<td>$2 \times 10^{-3}$</td>
</tr>
<tr>
<td>Period of oscillation of 3-cm microwaves</td>
<td>$1 \times 10^{-10}$</td>
</tr>
<tr>
<td>Typical period of rotation of a molecule</td>
<td>$1 \times 10^{-12}$</td>
</tr>
<tr>
<td>Shortest light pulse produced (1990)</td>
<td>$6 \times 10^{-15}$</td>
</tr>
<tr>
<td>Lifetime of least stable particles</td>
<td>$&lt;10^{-23}$</td>
</tr>
</tbody>
</table>

* Approximate values.

1-3 THE STANDARD OF TIME

The measurement of time has two aspects. For civil and for some scientific purposes we want to know the time of day so that we can order events in sequence. In most scientific work we want to know how long an event lasts (the time interval). Thus any time standard must be able to answer the questions “At what time does it occur?” and “How long does it last?” Table 3 shows the range of time intervals that can be measured. They vary by a factor of about $10^{63}$.

We can use any phenomenon that repeats itself as a measure of time. The measurement consists of counting the repetitions, including the fractions thereof. We could use an oscillating pendulum, a mass–spring system, or a quartz crystal, for example. Of the many repetitive phenomena in nature the rotation of the Earth on its axis, which determines the length of the day, was used as a time standard for centuries. One (mean solar) second was defined to be 1/86,400 of a (mean solar) day.

Quartz crystal clocks based on the electrically sustained periodic vibrations of a quartz crystal serve well as secondary time standards. A quartz clock can be calibrated against the rotating Earth by astronomical observations and used to measure time in the laboratory. The best of these have kept time for a year with a maximum accumulated error of 5 μs, but even this precision is not sufficient for modern science and technology.

To meet the need for a better time standard, atomic clocks have been developed in several countries. Figure 1 shows such a clock, based on a characteristic frequency of the microwave radiation emitted by atoms of the element cesium. This clock, maintained at the National Institute of Standards and Technology, forms the basis in this country for Coordinated Universal Time (UTC), for which time signals are available by shortwave radio (stations WWV and WWVH) and by telephone.

Figure 2 shows, by comparison with a cesium clock, variations in the rate of rotation of the Earth over a 4-year period.
period. These data show what a poor time standard the Earth's rotation provides for precise work. The variations that we see in Fig. 2 can be ascribed to tidal effects caused by the Moon and seasonal variations in the atmospheric winds.

The second based on the cesium clock was adopted as the international standard by the 13th General Conference on Weights and Measures in 1967. The following definition was given:

One second is the time occupied by $9,192,631,770$ vibrations of the radiation (of a specified wavelength) emitted by a cesium atom.

Two modern cesium clocks could run for 300,000 years before their readings would differ by more than 1 s. Hydrogen maser clocks have achieved the incredible precision of 1 s in 30,000,000 years. Clocks based on a single trapped atom may be able to improve on this precision by as much as 3 orders of magnitude. Figure 3 shows the impressive record of improvements in timekeeping that have occurred over the past 300 years or so, starting with the pendulum clock, invented by Christian Huygens in 1656, and ending with today's hydrogen maser.

1-4 THE STANDARD OF LENGTH*

The first international standard of length was a bar of a platinum-iridium alloy called the standard meter, which was kept at the International Bureau of Weights and Measures near Paris. The distance between two fine lines engraved near the ends of the bar, when the bar was held at a temperature of 0°C and supported mechanically in a prescribed way, was defined to be one meter. Historically, the meter was intended to be one ten-millionth of the distance from the north pole to the equator along the meridian line through Paris. However, accurate measure-

ments showed that the standard meter bar differs slightly (about 0.023%) from this value.

Because the standard meter is not very accessible, accurate master copies of it were made and sent to standardizing laboratories throughout the world. These secondary standards were used to calibrate other, still more accessible, measuring rods. Thus, until recently, every measuring rod or device derived its authority from the standard meter through a complicated chain of comparisons using microscopes and dividing engines. Since 1959 this statement had also been true for the yard, whose legal definition in the United States was adopted in that year to be

\[ 1 \text{ yard} = 0.9144 \text{ meter} \quad (\text{exactly}) \]

which is equivalent to

\[ 1 \text{ inch} = 2.54 \text{ centimeters} \quad (\text{exactly}). \]

The accuracy with which the necessary intercomparisons of length can be made by the technique of comparing fine scratches using a microscope is no longer satisfactory for modern science and technology. A more precise and reproducible standard of length was obtained when the American physicist Albert A. Michelson in 1893 compared the length of the standard meter with the wavelength of the red light emitted by atoms of cadmium. Michelson carefully measured the length of the meter bar and found that the standard meter was equal to 1,553,163.5 of those wavelengths. Identical cadmium lamps could easily be obtained in any laboratory, and thus Michelson found a way for scientists around the world to have a precise standard of length without relying on the standard meter bar.

Despite this technological advance, the metal bar remained the official standard until 1960, when the 11th General Conference on Weights and Measures adopted an atomic standard for the meter. The wavelength in vacuum of a certain orange-red light emitted by atoms of a particular isotope of krypton*, \(^{86}\text{Kr}\), in electrical discharge was chosen (see Fig. 4). Specifically, one meter was defined to be 1,650,763.73 wavelengths of this light. With the ability to make length measurements to a fraction of a wavelength, scientists could use this new standard to make comparisons of lengths to a precision below 1 part in \(10^9\).

The choice of an atomic standard offers advantages other than increased precision in length measurements. The \(^{86}\text{Kr}\) atoms are available everywhere, are identical, and emit light of the same wavelength. The particular wavelength chosen is uniquely characteristic of \(^{86}\text{Kr}\) and is sharply defined. The isotope can readily be obtained in pure form.

By 1983, the demands for higher precision had reached such a point that even the \(^{86}\text{Kr}\) standard could not meet them and in that year a bold step was taken. The meter was redefined as the distance traveled by a light wave in a specified time interval. In the words of the 17th General Conference on Weights and Measures:

*The meter is the length of the path traveled by light in vacuum during a time interval of \(1/299,792,458\) of a second.*

This is equivalent to saying that the speed of light \(c\) is now defined as

\[ c = 299,792,458 \text{ m/s} \quad (\text{exactly}). \]

*The superscript 86 in \(^{86}\text{Kr}\) gives the mass number (the number of protons plus neutrons in the nucleus) of this isotope of krypton. Naturally occurring krypton gas contains isotopes with mass numbers 78, 80, 82, 83, 84, and 86. The wavelength of the chosen radiation will differ in these different isotopes by about 1 part in \(10^9\), which is unacceptably large compared with the precision of the standard, about 1 part in \(10^9\). In the case of the cesium clock, there is only one naturally occurring isotope of cesium, which has mass number 133.*
This new definition of the meter was necessary because measurements of the speed of light had become so precise that the reproducibility of the \(^{85}\text{Kr}\) meter itself became the limiting factor. In view of this, it then made sense to adopt the speed of light as a defined quantity and to use it along with the precisely defined standard of time (the second) to redefine the meter.

Table 4 shows the range of measured lengths that can be compared with the standard.

**TABLE 4 SOME MEASURED LENGTHS**

<table>
<thead>
<tr>
<th>Length</th>
<th>Meters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance to farthest observed quasar</td>
<td>(2 \times 10^{26})</td>
</tr>
<tr>
<td>Distance to the Andromeda galaxy</td>
<td>(2 \times 10^{22})</td>
</tr>
<tr>
<td>Radius of our galaxy</td>
<td>(6 \times 10^{19})</td>
</tr>
<tr>
<td>Distance to the nearest star (Proxima Centauri)</td>
<td>(4 \times 10^{16})</td>
</tr>
<tr>
<td>Mean orbit radius for most distant planet (Pluto)</td>
<td>(6 \times 10^{12})</td>
</tr>
<tr>
<td>Radius of the Sun</td>
<td>(7 \times 10^{8})</td>
</tr>
<tr>
<td>Radius of the Earth</td>
<td>(6 \times 10^{6})</td>
</tr>
<tr>
<td>Height of Mt. Everest</td>
<td>(9 \times 10^{3})</td>
</tr>
<tr>
<td>Height of a typical person</td>
<td>(2 \times 10^{9})</td>
</tr>
<tr>
<td>Thickness of a page in this book</td>
<td>(1 \times 10^{-4})</td>
</tr>
<tr>
<td>Size of a typical virus</td>
<td>(1 \times 10^{-6})</td>
</tr>
<tr>
<td>Radius of a hydrogen atom</td>
<td>(5 \times 10^{-11})</td>
</tr>
<tr>
<td>Effective radius of a proton</td>
<td>(1 \times 10^{-15})</td>
</tr>
</tbody>
</table>

\(^{a}\) Approximate values.

**Sample Problem 2** A light-year is a measure of length (not a measure of time) equal to the distance that light travels in 1 year. Compute the conversion factor between light-years and meters, and find the distance to the star Proxima Centauri (\(4.0 \times 10^{16}\) m) in light-years.

**Solution** The conversion factor from years to seconds is

\[
1\ y = 1\ y \times \frac{365.25\ d}{y} \times \frac{24\ h}{d} \times \frac{60\ min}{h} \times \frac{60\ s}{min} = 3.16 \times 10^7\ s.
\]

The speed of light is, to three significant figures, \(3.00 \times 10^8\) m/s. Thus in 1 year, light travels a distance of

\[
(3.00 \times 10^8\ m/s) (3.16 \times 10^7\ s) = 9.48 \times 10^{15}\ m,
\]

so that

\[
1\ \text{light-year} = 9.48 \times 10^{15}\ m.
\]

The distance to Proxima Centauri is

\[
(4.0 \times 10^{16}\ m) \times \frac{1\ \text{light-year}}{9.48 \times 10^{15}\ m} = 4.2\ \text{light-years}.
\]

Light from Proxima Centauri thus takes about 4.2 years to travel to Earth.
The SI standard of mass is a platinum–iridium cylinder kept at the International Bureau of Weights and Measures and assigned, by international agreement, a mass of 1 kilogram. Secondary standards are sent to standardizing laboratories in other countries and the masses of other bodies can be found by an equal-arm balance technique to a precision of 1 part in $10^8$.

The U.S. copy of the international standard of mass, known as Prototype Kilogram No. 20, is housed in a vault at the National Institute of Standards and Technology (see Fig. 5). It is removed no more than once a year for checking the values of tertiary standards. Since 1889 Prototype No. 20 has been taken to France twice for recomparison with the master kilogram. When it is removed from the vault two people are always present, one to carry the kilogram in a pair of forceps, the second to catch the kilogram if the first person should fall.

Table 5 shows some measured masses. Note that they vary by a factor of about $10^{23}$. Most masses have been measured in terms of the standard kilogram by indirect methods. For example, we can measure the mass of the Earth (see Section 16-3) by measuring in the laboratory the gravitational force of attraction between two lead spheres and comparing it with the attraction of the Earth for a known mass. The masses of the spheres must be known by direct comparison with the standard kilogram.

On the atomic scale we have a second standard of mass, which is not an SI unit. It is the mass of the $^{12}$C atom which, by international agreement, has been assigned an atomic mass of 12 unified atomic mass units (abbreviation u), exactly and by definition. We can find the masses of other atoms to considerable accuracy by using a mass spectrometer (Fig. 6; see also Section 34-2). Table 6 shows some selected atomic masses, including the estimated uncertainties of measurement. We need a second standard of mass because present laboratory techniques permit us to compare atomic masses with each other to greater precision than we can presently compare them with the standard kilogram. However, development of an atomic mass standard to replace the standard kilogram is well under way. The relationship between the present atomic standard and the primary standard is approximately

$$1 \text{ u} = 1.661 \times 10^{-27} \text{ kg}.$$ 

A related SI unit is the mole, which measures the quantity of a substance. One mole of $^{12}$C atoms has a mass of exactly 12 grams and contains a number of atoms numerically equal to the Avogadro constant $N_A$:

$$N_A = 6.0221367 \times 10^{23} \text{ per mole.}$$

This is an experimentally determined number, with an uncertainty of about one part in a million. One mole of any other substance contains the same number of elementary entities (atoms, molecules, or whatever). Thus 1 mole of helium gas contains $N_A$ atoms of He, 1 mole of oxygen contains $N_A$ molecules of $O_2$, and 1 mole of water contains $N_A$ molecules of $H_2O$.

To relate an atomic unit of mass to a bulk unit, it is necessary to use the Avogadro constant. Replacing the standard kilogram with an atomic standard will require an improvement of at least two orders of magnitude in the precision of the measured value of $N_A$ to obtain masses with precisions of 1 part in $10^8$.

### TABLE 5 SOME MEASURED Masses

<table>
<thead>
<tr>
<th>Object</th>
<th>Kilograms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Known universe (estimate)</td>
<td>$10^{23}$</td>
</tr>
<tr>
<td>Our galaxy</td>
<td>$2 \times 10^{20}$</td>
</tr>
<tr>
<td>Sun</td>
<td>$2 \times 10^{30}$</td>
</tr>
<tr>
<td>Earth</td>
<td>$6 \times 10^{24}$</td>
</tr>
<tr>
<td>Moon</td>
<td>$7 \times 10^{22}$</td>
</tr>
<tr>
<td>Ocean liner</td>
<td>$7 \times 10^{17}$</td>
</tr>
<tr>
<td>Elephant</td>
<td>$4 \times 10^{13}$</td>
</tr>
<tr>
<td>Person</td>
<td>$6 \times 10^{11}$</td>
</tr>
<tr>
<td>Grape</td>
<td>$3 \times 10^{10}$</td>
</tr>
<tr>
<td>Speck of dust</td>
<td>$7 \times 10^{10}$</td>
</tr>
<tr>
<td>Virus</td>
<td>$1 \times 10^{-15}$</td>
</tr>
<tr>
<td>Penicillin molecule</td>
<td>$5 \times 10^{-17}$</td>
</tr>
<tr>
<td>Uranium atom</td>
<td>$4 \times 10^{-26}$</td>
</tr>
<tr>
<td>Proton</td>
<td>$2 \times 10^{-27}$</td>
</tr>
<tr>
<td>Electron</td>
<td>$9 \times 10^{-31}$</td>
</tr>
</tbody>
</table>

*a Approximate values.*
TABLE 6 SOME MEASURED ATOMIC MASSES

<table>
<thead>
<tr>
<th>Isotope</th>
<th>Mass (u)</th>
<th>Uncertainty (u)</th>
</tr>
</thead>
<tbody>
<tr>
<td>¹H</td>
<td>1.00782504</td>
<td>0.00000001</td>
</tr>
<tr>
<td>¹²C</td>
<td>12.00000000</td>
<td>(exact)</td>
</tr>
<tr>
<td>⁶⁴Cu</td>
<td>63.9297656</td>
<td>0.0000017</td>
</tr>
<tr>
<td>¹⁰⁴Ag</td>
<td>101.91195</td>
<td>0.00012</td>
</tr>
<tr>
<td>¹³⁷Cs</td>
<td>136.907073</td>
<td>0.000006</td>
</tr>
<tr>
<td>¹⁹⁰Pt</td>
<td>189.959917</td>
<td>0.000007</td>
</tr>
<tr>
<td>²³⁹Pu</td>
<td>238.0495546</td>
<td>0.0000024</td>
</tr>
</tbody>
</table>

1-6 PRECISION AND SIGNIFICANT FIGURES

As we improve the quality of our measuring instruments and the sophistication of our techniques, we can carry out experiments at ever increasing levels of precision; that is, we can extend the measured results to more and more significant figures and correspondingly reduce the experimental uncertainty of the result. Both the number of significant figures and the uncertainty tell something about our estimate of the precision of the result. That is, the result $x = 3\, m$ implies that we know less about $x$ than the value $x = 3.14159\, m$. When we declare $x = 3\, m$, we mean that we are reasonably certain that $x$ lies between 2\, m and 4\, m, while expressing $x$ as 3.14159\, m means that $x$ probably lies between 3.14158\, m and 3.14160\, m. If you express $x$ as 3\, m when in fact you really believe that $x$ is 3.14159\, m, you are withholding information that might be important. On the other hand, if you express $x = 3.14159\, m$ when you really have no basis for knowing anything other than $x = 3\, m$, you are being somewhat dishonest by claiming to have more information than you really do. Attention to significant figures is important when presenting the results of measurements and calculations, and it is equally as wrong to include too many as too few.

There are a few simple rules to follow in deciding how many significant figures to keep:

**Rule 1** Counting from the left and ignoring leading zeros, keep all digits up to the first doubtful one. That is, $x = 3\, m$ has only one significant figure, and expressing this value as $x = 0.003\, km$ does not change the number of significant figures. If we instead wrote $x = 3.0\, m$ (or, equivalently, $x = 0.0030\, km$), we would imply that we know the value of $x$ to two significant figures. In particular, don’t write down all 9 or 10 digits of your calculator display if they are not justified by the precision of the input data! Most calculations in this text are done with two or three significant figures.

Be careful about ambiguous notations: $x = 300\, m$ does not indicate whether there are one, two, or three significant figures; we don’t know whether the zeros are carrying information or merely serving as place holders. Instead, we should write $x = 3 \times 10^2$ or $3.0 \times 10^2$ or $3.00 \times 10^2$ to specify the precision more clearly.

**Rule 2** When multiplying or dividing, keep a number of significant figures in the product or quotient no greater than the number of significant figures in the least precise of the factors. Thus

$$2.3 \times 3.14159 = 7.2.$$
A bit of good judgment is occasionally necessary when applying this rule:

\[ 9.8 \times 1.03 = 10.1 \]

because, even though 9.8 technically has only two significant figures, it is very close to being a number with three significant figures. The product should therefore be expressed with three significant figures.

**Rule 3**  In adding or subtracting, the least significant digit of the sum or difference occupies the same relative position as the least significant digit of the quantities being added or subtracted. In this case the number of significant figures is not important; it is the position that matters. For example, suppose we wish to find the total mass of three objects as follows:

\[
\begin{align*}
103.9 \text{ kg} \\
2.10 \text{ kg} \\
0.319 \text{ kg}
\end{align*}
\]

The least significant or first doubtful digit is shown in **boldface**. By rule 1, we should include only one doubtful digit; thus the result should be expressed as 106.3 kg, for if the “3” is doubtful, then the following “19” gives no information and is useless.

---

**Sample Problem 3**  You wish to weigh your pet cat, but all you have available is an ordinary home platform scale. It is a digital scale, which displays your weight in a whole number of pounds. You therefore use the following scheme: you determine your own weight to be 119 pounds, and then holding the cat you find your combined weight to be 128 pounds. What is the fractional or percentage uncertainty in your weight and in the weight of your cat?

**Solution**  The least significant digit is the units digit, and so your weight is uncertain by about one pound. That is, your scale would read 119 lb for any weight between 118.5 and 119.5 lb. The fractional uncertainty is therefore

\[
\frac{1 \text{ lb}}{119 \text{ lb}} = 0.008 \quad \text{or} \quad 0.8\%.
\]

The weight of the cat is 128 lb − 119 lb = 9 lb. However, the uncertainty in the cat’s weight is still about 1 lb, and so the fractional uncertainty is

\[
\frac{1 \text{ lb}}{9 \text{ lb}} = 0.11 = 11\%.
\]

Although the absolute uncertainty in your weight and the cat’s weight is the same (1 lb), the relative uncertainty in your weight is an order of magnitude smaller than the relative uncertainty in the cat’s weight. If you tried to weigh a 1-lb kitten by this method, the relative uncertainty in its weight would be 100%. This illustrates a commonly occurring danger in the subtraction of two numbers that are nearly equal: the relative or percentage uncertainty in the difference can be very large.

---

**1-7 DIMENSIONAL ANALYSIS**

Associated with every measured or calculated quantity is a **dimension**. For example, both the absorption of sound by an enclosure and the probability for nuclear reactions to occur have the dimensions of an area. The units in which the quantities are expressed do not affect the dimension of the quantities: an area is still an area whether it is expressed in m² or ft² or acres or sabins (sound absorption) or barns (nuclear reactions).

Just as we defined our measurement standards earlier in this chapter as fundamental quantities, we can choose a set of fundamental dimensions based on independent measurement standards. For mechanical quantities, mass, length, and time are elementary and independent, so they can serve as fundamental dimensions. They are represented respectively by M, L, and T.

Any equation must be **dimensionally consistent**; that is, the dimensions on both sides must be the same. Attention to dimensions can often keep you from making errors in writing equations. For example, the distance x covered in a time t by an object starting from rest and moving subject to a constant acceleration a will be shown in the next chapter to be \( x = \frac{1}{2} at^2 \). Acceleration is measured in units such as m/s². We use square brackets [ ] to denote “the dimension of,” so that \([x] = \text{L} \) or \([t] = \text{T} \). It follows that \([a] = \text{LT}^{-2} \) or \(\text{LT}^{-2} \). Keeping the units, and therefore the dimension, of acceleration in mind, you will therefore never be tempted to write \( x = \frac{1}{2} at \) or \( x = \frac{1}{4} at^2 \).

The analysis of dimensions can often help in working out equations. The following two sample problems illustrate this procedure.

**Sample Problem 4**  To keep an object moving in a circle at constant speed requires a force called the “centripetal force.” (Circular motion is discussed in Chapter 4.) Do a dimensional analysis of the centripetal force.

**Solution**  We begin by asking “On which mechanical variables could the centripetal force \( F \) depend?” The moving object has only three properties that are likely to be important: its mass \( m \), its speed \( v \), and the radius \( r \) of its circular path. The centripetal force \( F \) must therefore be given, apart from any dimensionless constants, by an equation of the form

\[
F \propto m^a v^b r^c
\]

where the symbol \( \propto \) means “is proportional to,” and where \( a, b, \) and \( c \) are numerical exponents to be determined from analyzing the dimensions. As we wrote in Section 1-2 (and as we shall
discuss in Chapter 5), force has units of \( \text{kg} \cdot \text{m/s}^2 \), and therefore its dimensions are \( [F] = \text{MLT}^{-2} \). We can therefore write the centripetal force equation in terms of dimensions as

\[
[F] = [m^a] \cdot [v^b] \cdot [r^c] \\
\text{MLT}^{-2} = M^a (L/T)^b L^c \\
= M^a L^{b+c} T^{-b}.
\]

Dimensional consistency means that the fundamental dimensions must be the same on each side. Thus, equating the exponents,

- exponents of \( M \): \( a = 1 \);
- exponents of \( T \): \( b = 2 \);
- exponents of \( L \): \( b + c = 1 \) so \( c = -1 \).

The resulting expression is

\[
F \propto \frac{mv^2}{r}.
\]

The actual expression for centripetal force, derived from Newton’s laws and the geometry of circular motion, is \( F = \frac{mv^2}{r} \). The dimensional analysis gives us the exact dependence on the mechanical variables! This is really a happy accident, because dimensional analysis can’t tell us anything about constants that do not have dimensions. In this case the constant happens to be 1.

**Solution** Using the units given for the three constants, we can obtain their dimensions:

\[
[c] = [\text{m/s}] = \text{LT}^{-1} \\
[G] = [\text{m}^3/\text{s}^2 \cdot \text{kg}] = \text{L}^3 \text{T}^{-2} \text{M}^{-1} \\
[h] = [\text{kg} \cdot \text{m}^2/\text{s}] = \text{ML}^2 \text{T}^{-1}
\]

Let the Planck time depend on these constants as

\[
t_P \propto c^i G^j h^k,
\]

where \( i, j, \) and \( k \) are exponents to be determined. The dimensions of this expression are

\[
[t_P] = [c^i] \cdot [G^j] \cdot [h^k] \\
T = (\text{LT}^{-1})^i (\text{L}^3 \text{T}^{-2} \text{M}^{-1})^j (\text{ML}^2 \text{T}^{-1})^k \\
= L^{i+3j+2k} T^{-i-2j-k} M^{-j+k}.
\]

Equating powers on both sides gives

- exponents of \( L \): \( 0 = i + 3j + 2k \)
- exponents of \( T \): \( 1 = -i - 2j - k \)
- exponents of \( M \): \( 0 = -j + k \)

and solving these three equations for the three unknowns, we find

\[
i = -\frac{3}{2}, \quad j = \frac{1}{2}, \quad k = \frac{1}{2}.
\]

Thus

\[
t_P \propto c^{-3/2} G^{1/2} h^{1/2} \frac{Gh}{c^3} = \sqrt{\frac{(6.67 \times 10^{-11} \text{ m}^3/\text{s}^2 \cdot \text{kg})(6.63 \times 10^{-34} \text{ kg} \cdot \text{m}^2/\text{s})}{(3.00 \times 10^8 \text{ m/s})^3}} = 1.35 \times 10^{-43} \text{ s}.
\]

As commonly defined, the Planck time differs from this value by a factor of \((2\pi)^{-1/2}\). Such dimensionless factors cannot be found by this technique.

In similar fashion, we can determine the Planck length and the Planck mass, which also have very fundamental interpretations (see Problems 41 and 42).

---

**QUESTIONS**

1. How would you criticize this statement: “Once you have picked a standard, by the very meaning of ‘standard’ it is invariable”?
2. List characteristics other than accessibility and invariability that you would consider desirable for a physical standard.
3. Can you imagine a system of base units (Table 1) in which time was not included?
4. Of the seven base units listed in Table 1, only one — the kilogram — has a prefix (see Table 2). Would it be wise to redefine the mass of that platinum-iridium cylinder at the International Bureau of Weights and Measures as 1 g rather than 1 kg?
5. What does the prefix “micro-” signify in the words “microwave oven”? It has been proposed that food that has been irradiated by gamma rays to lengthen its shelf life be marked “picowaved.” What do you suppose that means?
6. Many capable investigators, on the evidence, believe in the reality of extrasensory perception. Assuming that ESP is...
indeed a fact of nature, what physical quantity or quantities would you seek to define to describe this phenomenon quantitatively?

7. According to a point of view adopted by some physicists and philosophers, if we cannot describe procedures for determining a physical quantity, we say that the quantity is undetectable and should be given up as having no physical reality. Not all scientists accept this view. What in your opinion are the merits and drawbacks of this point of view?

8. Name several repetitive phenomena occurring in nature that could serve as reasonable time standards.

9. You could define “1 second” to be one pulse beat of the current president of the American Association of Physics Teachers. Galileo used his pulse as a timing device in some of his work. Why is a definition based on the atomic clock better?

10. What criteria should be satisfied by a good clock?

11. From what you know about pendulums, cite the drawbacks to using the period of a pendulum as a time standard.

12. On June 30, 1981, the minute extending from 10:59 to 11:00 a.m. was arbitrarily lengthened to contain 61 s. The last day of 1989 also was lengthened by 1 s. Such a leap second is occasionally introduced to compensate for the fact that, as measured by our atomic time standard, the Earth’s rotation rate is slowly decreasing. Why is it desirable to readjust our clocks in this way?

13. A radio station advertises “at 89.5 on your FM dial.” What does this number mean?

14. Why are there no SI base units for area or volume?

15. The meter was originally intended to be one ten-millionth of the meridian line from the north pole to the equator that passes through Paris. This definition disagrees with the standard meter bar by 0.023%. Does this mean that the standard meter bar is inaccurate to this extent?

16. Can length be measured along a curved line? If so, how?

17. When the meter bar was taken to be the standard of length, its temperature was specified. Can length be called a fundamental property if another physical quantity, such as temperature, must be specified in choosing a standard?

18. In redefining the meter in terms of the speed of light, why did the delegates to the 1983 General Conference on Weights and Measures not simplify matters by defining the speed of light to be $3 \times 10^8$ m/s exactly? For that matter, why did they not define it to be 1 m/s exactly? Were both of these possibilities open to them? If so, why did they reject them?

19. Suggest a way to measure (a) the radius of the Earth, (b) the distance between the Sun and the Earth, and (c) the radius of the Sun.

20. Suggest a way to measure (a) the thickness of a sheet of paper, (b) the thickness of a soap bubble film, and (c) the diameter of an atom.

21. If someone told you that every dimension of every object had shrunk to half its former value overnight, how could you refute this statement?

22. Is the current standard kilogram of mass accessible, invariable, reproducible, and indestructible? Does it have simplicity for comparison purposes? Would an atomic standard be better in any respect? Why don’t we adopt an atomic standard, as we do for length and time?

23. Why do we find it useful to have two standards of mass, the kilogram and the $^{13}$C atom?

24. How does one obtain the relation between the masses of the standard kilogram and the mass of the $^{13}$C atom?

25. Suggest practical ways by which one could determine the masses of the various objects listed in Table 5.

26. Suggest objects whose masses would fall in the wide range in Table 5 between that of an ocean liner and the Moon and estimate their masses.

27. Critics of the metric system often cloud the issue by saying things such as: “Instead of buying 1 lb of butter you will have to ask for 0.454 kg of butter.” The implication is that life would be more complicated. How might you refute this?

---

**PROBLEMS**

**Section 1-2 The International System of Units**

1. Use the prefixes in Table 2 and express (a) $10^6$ phones; (b) $10^{-6}$ phones; (c) $10^1$ cards; (d) $10^9$ lows; (e) $10^{12}$ bulls; (f) $10^{-1}$ mates; (g) $10^{-2}$ pedes; (h) $10^{-9}$ Nannettes; (i) $10^{-12}$ boos; (j) $10^{-18}$ boys; (k) $2 \times 10^2$ withis; (l) $2 \times 10^2$ mockingbirds. Now that you have the idea, invent a few more similar expressions. (See p. 61 of *A Random Walk in Science*, compiled by R. L. Weber; Crane, Russak & Co., New York, 1974.)

2. Some of the prefixes of the SI units have crept into everyday language. (a) What is the weekly equivalent of an annual salary of $36K$ (= 36 k$S$)? (b) A lottery awards 10 megabucks as the top prize, payable over 20 years. How much is received in each monthly check? (c) The hard disk of a computer has a capacity of 30 MB (= 30 megabytes). At 8 bytes/word, how many words can it store? In computerese, kilo means 1024 (= $2^{10}$), not 1000.

**Section 1-3 The Standard of Time**

3. Enrico Fermi once pointed out that a standard lecture period (50 min) is close to 1 microcentury. How long is a microcentury in minutes, and what is the percent difference from Fermi’s approximation?

4. New York and Los Angeles are about 3000 mi apart; the time difference between these two cities is 3 h. Calculate the circumference of the Earth.
5. A convenient substitution for the number of seconds in a year is $\pi \times 10^7$. To within what percentage error is this correct?

6. Shortly after the French Revolution, as part of their introduction of the metric system, the revolutionary National Convention made an attempt to introduce decimal time. In this plan, which was not successful, the day—starting at midnight—was divided into 10 decimal hours consisting of 100 decimal minutes each. The hands of a surviving decimal pocket watch are stopped at 8 decimal hours, 22.8 decimal minutes. What time is it? See Fig. 7.

![Figure 7 Problem 6.](image)

7. (a) A unit of time sometimes used in microscopic physics is the shake. One shake equals $10^{-8}$ s. Are there more shakes in a second than there are seconds in a year? (b) Humans have existed for about $10^6$ years, whereas the universe is about $10^{10}$ years old. If the age of the universe is taken to be 1 day, for how many seconds have humans existed?

8. In two different track meets, the winners of the mile race ran their races in 3 min 58.05 s and 3 min 58.20 s. In order to conclude that the runner with the shorter time was indeed faster, what is the maximum tolerable error, in feet, in laying out the distances?

9. A certain pendulum clock (with a 12-h dial) happens to gain 1 min/day. After setting the clock to the correct time, how long must one wait until it again indicates the correct time?

10. Five clocks are being tested in a laboratory. Exactly at noon, as determined by the WWV time signal, on the successive days of a week the clocks read as follows:

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>11:59:59</td>
<td>12:00:02</td>
<td>11:59:57</td>
<td>12:00:07</td>
<td>12:00:02</td>
<td>11:59:56</td>
<td>12:00:03</td>
</tr>
<tr>
<td>E</td>
<td>12:03:59</td>
<td>12:02:49</td>
<td>12:01:54</td>
<td>12:01:52</td>
<td>12:01:32</td>
<td>12:01:22</td>
<td>12:01:12</td>
</tr>
</tbody>
</table>

How would you arrange these five clocks in the order of their relative value as good timekeepers? Justify your choice.

11. The age of the universe is about $5 \times 10^7$ s; the shortest light pulse produced in a laboratory (1990) lasted for only $6 \times 10^{-15}$ s (see Table 3). Identify a physically meaningful time interval approximately halfway between these two on a logarithmic scale.

12. Assuming that the length of the day uniformly increases by 0.001 s in a century, calculate the cumulative effect on the measure of time over 20 centuries. Such a slowing down of the Earth’s rotation is indicated by observations of the occurrences of solar eclipses during this period.

13. The time it takes the Moon to return to a given position as seen against the background of fixed stars, 27.3 days, is called a sidereal month. The time interval between identical phases of the Moon is called a lunar month. The lunar month is longer than a sidereal month. Why and by how much?

**Section 1-4 The Standard of Length**

14. Your French pen pal Pierre writes to say that he is 1.9 m tall. What is his height in British units?

15. (a) In track meets both 100 yards and 100 meters are used as distances for dashes. Which is longer? By how many meters is it longer? By how many feet? (b) Track and field records are kept for the mile and the so-called metric mile (1500 meters). Compare these distances.

16. The stability of the cesium clock used as an atomic time standard is such that two cesium clocks would gain or lose 1 s with respect to each other in about 300,000 y. If this same precision were applied to the distance between New York and San Francisco (2572 mi), by how much would successive measurements of this distance tend to differ?

17. Antarctica is roughly semicircular in shape with a radius of 2000 km. The average thickness of the ice cover is 3000 m. How many cubic centimeters of ice does Antarctica contain? (Ignore the curvature of the Earth.)

18. A unit of area, often used in expressing areas of land, is the hectare, defined as $10^4$ m$^2$. An open-pit coal mine consumes 77 hectares of land, down to a depth of 26 m, each year. What volume of earth, in cubic kilometers, is removed in this time?

19. The Earth is approximately a sphere of radius $6.37 \times 10^6$ m. (a) What is its circumference in kilometers? (b) What is its surface area in square kilometers? (c) What is its volume in cubic kilometers?

20. The approximate maximum speeds of various animals follows, but in different units of speed. Convert these data to m/s, and thereby arrange the animals in order of increasing maximum speed: squirrel, 19 km/h; rabbit, 30 knots; snail,
0.030 mi/h; spider, 1.8 ft/s; cheetah, 1.9 km/min; human, 1000 cm/s; fox, 1100 m/min; lion, 1900 km/day.

21. A certain spaceship has a speed of 19,200 mi/h. What is its speed in light-years per century?

22. A new car is equipped with a “real-time” dashboard display of fuel consumption. A switch permits the driver to toggle back and forth between British units and SI units. However, the British display shows mi/gal while the SI version is the inverse, L/km. What SI reading corresponds to 30.0 mi/gal?

23. Astronomical distances are so large compared to terrestrial ones that much larger units of length are used for easy comprehension of the relative distances of astronomical objects. An astronomical unit (AU) is equal to the average distance from Earth to the Sun, $1.50 \times 10^8$ km. A parsec (pc) is the distance at which 1 AU would subtend an angle of 1 second of arc. A light-year (ly) is the distance that light, traveling through a vacuum with a speed of $3.00 \times 10^5$ km/s, would cover in 1 year. (a) Express the distance from Earth to the Sun in parsecs and in light-years. (b) Express a light-year and a parsec in kilometers. Although the light-year is much used in popular writing, the parsec is the unit used professionally by astronomers.

24. The effective radius of a proton is about $1 \times 10^{-15}$ m; the radius of the observable universe (given by the distance to the farthest observable quasar) is $2 \times 10^{26}$ m (see Table 4). Identify a physically meaningful distance that is approximately halfway between these two extremes on a logarithmic scale.

25. The average distance of the Sun from Earth is 390 times the average distance of the Moon from Earth. Now consider a total eclipse of the Sun (Moon between Earth and Sun; see Fig. 8) and calculate (a) the ratio of the Sun's diameter to the Moon’s diameter, and (b) the ratio of the Sun’s volume to the Moon’s volume. (c) The angle intercepted at the eye by the Moon is 0.52° and the distance between Earth and the Moon is $3.82 \times 10^5$ km. Calculate the diameter of the Moon.

26. The navigator of the oil tanker Exxon Valdez uses the satellites of the Global Positioning System (GPS/NAVSTAR) to find latitude and longitude; see Fig. 9. These are 43°36’25.3”N and 77°31’48.2”W. If the accuracy of these determinations is ±0.5”, what is the uncertainty in the tanker’s position measured along (a) a north–south line (meridian of longitude) and (b) an east–west line (parallel of latitude)? (c) Where is the tanker?

Section 1-5 The Standard of Mass

27. Using conversions and data in the chapter, determine the number of hydrogen atoms required to obtain 1.00 kg of hydrogen.

28. One molecule of water (H₂O) contains two atoms of hydrogen and one atom of oxygen. A hydrogen atom has a mass of 1.0 u and an atom of oxygen has a mass of 16 u. (a) What is the mass in kilograms of one molecule of water? (b) How many molecules of water are in the oceans of the world? The oceans have a total mass of $1.4 \times 10^{21}$ kg.

29. In continental Europe, one “pound” is half a kilogram. Which is the better buy: one Paris pound of coffee for $3.00 or one New York pound of coffee for $2.40?

30. A room has dimensions of 21 ft × 13 ft × 12 ft. What is the mass of the air it contains? The density of air at room temperature and normal atmospheric pressure is 1.21 kg/m³.

31. A typical sugar cube has an edge length of 1 cm. If you had a cubical box that contained 1 mole of sugar cubes, what would its edge length be?

32. A person on a diet loses 2.3 kg (corresponding to about 5 lb) per week. Express the mass loss rate in milligrams per second.

33. Suppose that it takes 12 h to drain a container of 5700 m³ of water. What is the mass flow rate (in kg/s) of water from the container? The density of water is 1000 kg/m³.

34. The grains of fine California beach sand have an average radius of 50 μm. What mass of sand grains would have a total surface area equal to the surface area of a cube exactly 1 m on an edge? Sand is made of silicon dioxide, 1 m³ of which has a mass of 2600 kg.

35. The standard kilogram (see Fig. 5) is in the shape of a circular cylinder with its height equal to its diameter. Show that, for a circular cylinder of fixed volume, this equality gives the smallest surface area, thus minimizing the effects of surface contamination and wear.

36. The distance between neighboring atoms, or molecules, in a solid substance can be estimated by calculating twice the
radius of a sphere with volume equal to the volume per atom of the material. Calculate the distance between neighboring atoms in (a) iron and (b) sodium. The densities of iron and sodium are 7870 kg/m³ and 1013 kg/m³, respectively; the mass of an iron atom is $9.27 \times 10^{-26}$ kg, and the mass of a sodium atom is $3.82 \times 10^{-26}$ kg.

Section 1-6 Precision and Significant Figures

37. For the period 1960–1983, the meter was defined to be 1,650,763.73 wavelengths of a certain orange-red light emitted by krypton atoms. Compute the distance in nanometers corresponding to one wavelength. Express your result using the proper number of significant figures.

38. (a) Evaluate 37.76 + 0.132 to the correct number of significant figures. (b) Evaluate 16.264 − 16.26325 to the correct number of significant figures.

39. (a) A rectangular metal plate has a length of 8.43 cm and a width of 5.12 cm. Calculate the area of the plate to the correct number of significant figures. (b) A circular metal plate has a radius of 3.7 cm. Calculate the area of the plate to the correct number of significant figures.

Section 1-7 Dimensional Analysis

40. Porous rock through which groundwater can move is called an aquifer. The volume $V$ of water that, in time $t$, moves through a cross section of area $A$ of the aquifer is given by

$$\frac{V}{t} = KA \frac{H}{L},$$

where $H$ is the vertical drop of the aquifer over the horizontal distance $L$; see Fig. 10. This relation is called Darcy's law. The quantity $K$ is the hydraulic conductivity of the aquifer. What are the SI units of $K$?

41. In Sample Problem 5, the constants $h$, $G$, and $c$ were combined to obtain a quantity with the dimensions of time. Repeat the derivation to obtain a quantity with the dimensions of length, and evaluate the result numerically. Ignore any dimensionless constants. This is the Planck length, the size of the observable universe at the Planck time.

42. Repeat the procedure of Problem 41 to obtain a quantity with the dimensions of mass. This gives the Planck mass, the mass of the observable universe at the Planck time.