POTENCIAL ELÉCTRICO
\[ \Delta U = -q_0 \int_A^B \mathbf{E} \cdot d\mathbf{s} \]

Potencial eléctrico

\[ V = \frac{U}{q_0} \]

Potencial eléctrico

\[ \Delta V = \frac{\Delta U}{q_0} = -\int_A^B \mathbf{E} \cdot d\mathbf{s} \]

Diferencia de potencial entre dos puntos

\[ W = q \Delta V \]

Trabajo

\[ 1 \text{ V} = 1 \frac{\text{J}}{\text{C}} \]
Example 25.2 Motion of a Proton in a Uniform Electric Field

A proton is released from rest in a uniform electric field that has a magnitude of $8.0 \times 10^4 \text{ V/m}$ (Fig. 25.6). The proton undergoes a displacement of 0.50 m in the direction of $E$.

(A) Find the change in electric potential between points $A$ and $B$.

**Solution** Because the positively charged proton moves in the direction of the field, we expect it to move to a position of lower electric potential. From Equation 25.6, we have

$$\Delta V = -Ed = -(8.0 \times 10^4 \text{ V/m})(0.50 \text{ m}) = -4.0 \times 10^4 \text{ V}$$

(B) Find the change in potential energy of the proton-field system for this displacement.

**Solution** Using Equation 25.3,

$$\Delta U = q_0 \Delta V = e \Delta V$$

$$= (1.6 \times 10^{-19} \text{ C})(-4.0 \times 10^4 \text{ V})$$

$$= -6.4 \times 10^{-15} \text{ J}$$

The negative sign means the potential energy of the system decreases as the proton moves in the direction of the electric field. As the proton accelerates in the direction of the field, it gains kinetic energy and at the same time the system loses electric potential energy.

(C) Find the speed of the proton after completing the 0.50 m displacement in the electric field.

**Solution** The charge-field system is isolated, so the mechanical energy of the system is conserved:

$$\Delta K + \Delta U = 0$$

$$\left(\frac{1}{2}mv^2 - 0\right) + e \Delta V = 0$$

$$v = \sqrt{\frac{-(2e\Delta V)}{m}}$$

$$= \sqrt{\frac{-2(1.6 \times 10^{-19} \text{ C})(-4.0 \times 10^4 \text{ V})}{1.67 \times 10^{-27} \text{ kg}}}$$

$$= 2.8 \times 10^6 \text{ m/s}$$

**What If?** What if the situation is exactly the same as that shown in Figure 25.6, but no proton is present? Could both parts (A) and (B) of this example still be answered?
If the system consists of more than two charged particles, we can obtain the total potential energy by calculating $U$ for every pair of charges and summing the terms algebraically. As an example, the total potential energy of the system of three charges shown in Figure 25.11 is

$$U = k_e \left( \frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right)$$  \hspace{1cm} (25.14)$$

**Figure 25.11** Three point charges are fixed at the positions shown. The potential energy of this system of charges is given by Equation 25.14.
Example 25.3  The Electric Potential Due to Two Point Charges

A charge \( q_1 = 2.00 \, \mu C \) is located at the origin, and a charge \( q_2 = -6.00 \, \mu C \) is located at \( (0, 3.00) \) m, as shown in Figure 25.12a.

(A) Find the total electric potential due to these charges at the point \( P \), whose coordinates are \( (4.00, 0) \) m.

**Solution** For two charges, the sum in Equation 25.12 gives

\[
V_P = k_e \left( \frac{q_1}{r_1} + \frac{q_2}{r_2} \right)
\]

\[
V_P = (8.99 \times 10^9 \, \text{N} \cdot \text{m}^2/\text{C}^2) \times \left( \frac{2.00 \times 10^{-6} \, \text{C}}{4.00 \, \text{m}} - \frac{6.00 \times 10^{-6} \, \text{C}}{5.00 \, \text{m}} \right)
\]

\[
= -6.29 \times 10^3 \, \text{V}
\]

(B) Find the change in potential energy of the system of two charges plus a charge \( q_3 = 3.00 \, \mu C \) as the latter charge moves from infinity to point \( P \) (Fig. 25.12b).

**Solution** When the charge \( q_3 \) is at infinity, let us define \( U_i = 0 \) for the system, and when the charge is at \( P \), \( U_f = q_3 V_P \); therefore,

\[
\Delta U = q_3 V_P - 0 = (3.00 \times 10^{-6} \, \text{C})(-6.29 \times 10^3 \, \text{V})
\]

\[
= -1.89 \times 10^{-2} \, \text{J}
\]

Therefore, because the potential energy of the system has decreased, positive work would have to be done by an external agent to remove the charge from point \( P \) back to infinity.

**What If?** You are working through this example with a classmate and she says, “Wait a minute! In part (B), we ignored the potential energy associated with the pair of charges \( q_1 \) and \( q_2 \)” How would you respond?

**Answer** Given the statement of the problem, it is not necessary to include this potential energy, because part (B) asks for the change in potential energy of the system as \( q_3 \) is brought in from infinity. Because the configuration of charges \( q_1 \) and \( q_2 \) does not change in the process, there is no \( \Delta U \) associated with these charges. However, if part (B) had asked to find the change in potential energy when all three charges start out infinitely far apart and are then brought to the positions in Figure 25.12b, we would need to calculate the change as follows, using Equation 25.14:

\[
U = k_e \left( \frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right)
\]

\[
= (8.99 \times 10^9 \, \text{N} \cdot \text{m}^2/\text{C}^2) \times \left( \frac{(2.00 \times 10^{-6} \, \text{C})(-6.00 \times 10^{-6} \, \text{C})}{3.00 \, \text{m}} + \frac{(2.00 \times 10^{-6} \, \text{C})(3.00 \times 10^{-6} \, \text{C})}{4.00 \, \text{m}} + \frac{(3.00 \times 10^{-6} \, \text{C})(-6.00 \times 10^{-6} \, \text{C})}{5.00 \, \text{m}} \right)
\]

\[
= -5.48 \times 10^{-2} \, \text{J}
\]
Figure 25.12 (Example 25.3) (a) The electric potential at $P$ due to the two charges $q_1$ and $q_2$ is the algebraic sum of the potentials due to the individual charges. (b) A third charge $q_3 = 3.00 \, \mu \text{C}$ is brought from infinity to a position near the other charges.
Example 25.5  Electric Potential Due to a Uniformly Charged Ring

(A) Find an expression for the electric potential at a point $P$ located on the perpendicular central axis of a uniformly charged ring of radius $a$ and total charge $Q$.

Solution  Figure 25.16, in which the ring is oriented so that its plane is perpendicular to the $x$ axis and its center is at the origin, helps us to conceptualize this problem. Because the ring consists of a continuous distribution of charge rather than a set of discrete charges, we categorize this problem as one in which we need to use the integration technique represented by Equation 25.20. To analyze the problem, we take point $P$ to be at a distance $x$ from the center of the ring, as shown in Figure 25.16. The charge element $dq$ is at a distance $\sqrt{x^2 + a^2}$ from point $P$. Hence, we can express $V$ as

$$V = k_e \int \frac{dq}{r} = k_e \int \frac{dq}{\sqrt{x^2 + a^2}}$$

Because each element $dq$ is at the same distance from point $P$, we can bring $\sqrt{x^2 + a^2}$ in front of the integral sign, and $V$ reduces to

$$V = \frac{k_e}{\sqrt{x^2 + a^2}} \int dq = \frac{k_e Q}{\sqrt{x^2 + a^2}} \quad (25.21)$$

The only variable in this expression for $V$ is $x$. This is not surprising because our calculation is valid only for points along the $x$ axis, where $y$ and $z$ are both zero.

(B) Find an expression for the magnitude of the electric field at point $P$.

Solution  From symmetry, we see that along the $x$ axis $E$ can have only an $x$ component. Therefore, we can use Equation 25.16:

$$E_x = -\frac{dV}{dx} = -k_e Q \frac{d}{dx} \left( x^2 + a^2 \right)^{-1/2}$$

$$= -k_e Q \left( -\frac{1}{2} \right) (x^2 + a^2)^{-3/2} (2x)$$

$$E_x = \frac{k_e Q x}{(x^2 + a^2)^{3/2}} \quad (25.22)$$

To finalize this problem, we see that this result for the electric field agrees with that obtained by direct integration (see Example 23.8). Note that $E_x = 0$ at $x = 0$ (the center of the ring). Could you have guessed this?
Example 25.6  Electric Potential Due to a Uniformly Charged Disk

A uniformly charged disk has radius \( a \) and surface charge density \( \sigma \). Find

(A) the electric potential and

(B) the magnitude of the electric field along the perpendicular central axis of the disk.

Solution (A) Again, we choose the point \( P \) to be at a distance \( x \) from the center of the disk and take the plane of the disk to be perpendicular to the \( x \) axis. We can simplify the problem by dividing the disk into a series of charged rings of infinitesimal width \( dr \). The electric potential due to each ring is given by Equation 25.21. Consider one such ring of radius \( r \) and width \( dr \), as indicated in Figure 25.17. The surface area of the ring is \( dA = 2\pi r \, dr \). From the definition of surface charge density (see Section 23.5), we know that the charge on the ring is \( dq = \sigma \, dA = \sigma^2 \pi r \, dr \). Hence, the potential at the point \( P \) due to this ring is

\[
dV = \frac{k_e \, dq}{\sqrt{r^2 + x^2}} = \frac{k_e \sigma^2 \pi r \, dr}{\sqrt{r^2 + x^2}} \]

where \( x \) is a constant and \( r \) is a variable. To find the total electric potential at \( P \), we sum over all rings making up the disk. That is, we integrate \( dV \) from \( r = 0 \) to \( r = a \):

\[
V = \pi k_e \sigma \int_0^a \frac{2r \, dr}{\sqrt{r^2 + x^2}} = \pi k_e \sigma \int_0^a \frac{(r^2 + x^2)^{-1/2} \, 2r \, dr}{\sqrt{r^2 + x^2}}
\]

This integral is of the common form \( \int u^n \, du \) and has the value \( u^{n+1}/(n+1) \), where \( n = -\frac{1}{2} \) and \( u = r^2 + x^2 \). This gives

\[
V = \pi k_e \sigma \left[ \frac{(r^2 + x^2)^{1/2}}{1/2} \right]_0^a = \pi k_e \sigma \left[ \frac{2}{\sqrt{r^2 + x^2}} \right]_0^a.
\]

(B) As in Example 25.5, we can find the electric field at any axial point using Equation 25.16:

\[
E_x = -\frac{dV}{dx} = 2\pi k_e \sigma \left( 1 - \frac{x}{\sqrt{x^2 + a^2}} \right)
\]

The calculation of \( V \) and \( E \) for an arbitrary point off the axis is more difficult to perform, and we do not treat this situation in this text.

Figure 25.17  (Example 25.6) A uniformly charged disk of radius \( a \) lies in a plane perpendicular to the \( x \) axis. The calculation of the electric potential at any point \( P \) on the \( x \) axis is simplified by dividing the disk into many rings of radius \( r \) and width \( dr \), with area \( 2\pi r \, dr \).
The Millikan Oil-Drop Experiment

Active Figure 25.27  Schematic drawing of the Millikan oil-drop apparatus.
Figure 25.30 (a) Schematic diagram of an electrostatic precipitator. The high negative electric potential maintained on the central coiled wire creates a corona discharge in the vicinity of the wire. Compare the air pollution when the electrostatic precipitator is (b) operating and (c) turned off.
Xerography and Laser Printers

(a) Charging the drum
(b) Imaging the document
(c) Applying the toner
(d) Transferring the toner to the paper
(e) Laser printer drum

Light causes some areas of drum to become electrically conducting, removing positive charge.