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Fluidized-bed bioreactors

G. ANDREWS

Biotechnology Unit, Idaho National Engineering Laboratory, Idaho Falls, ID 83415, USA

Introduction

Large-scale biochemical processes involve three main costs: the capital cost of the bioreactor and associated equipment; the cost of the substrates, and the costs associated with separating and purifying the product. These costs are always present, although their relative importance differs considerably in different cases. In wastewater treatment for example, the cost of the 'substrate' can safely be ignored, and product separation is restricted to separating the biomass from the clean water. The reactor cost is therefore the dominant factor in the overall process economics. At the other extreme, in the manufacture of a therapeutic protein by tissue culture, the expensive media required and the high product purity requirements make the reactor cost relatively insignificant.

The objective of the biochemical engineer in designing and operating a bioreactor is to minimize the total costs of the process, and this usually requires compromise between the three different costs. To illustrate the point, consider the production of ethanol from corn hydrolysate. As corn is expensive it is important to minimize the substrate cost by obtaining a high conversion (fraction of the substrate fermented) and a high product yield (gram ethanol produced per gram of substrate consumed). Unfortunately, due to the effect of product inhibition, a high conversion requires a long fermentation time which means that large, expensive reactors are required for a given rate of ethanol production. Product inhibition could be reduced simply by diluting the substrate in order to reduce the final product concentration, but this would increase the cost of product separation. Similarly, because cell growth consumes substrate that could otherwise be used to make product, a high product yield could be achieved with non-growing cells. However, the low metabolic rate of non-growing cells would again require long fermentation times and large, expensive reactors. Note that the compromise between conversion, yield, reaction rate and final product concentration that was optimal for corn hydrolysate would be quite wrong for a cheap substrate such as cheese whey.
It is the need for these compromises that makes correct bioreactor design so important and that has motivated the development of the fluidized-bed bioreactor. The purpose of this review is to explain this development in terms of the advantages of the fluidized bed over other types of reactors. Emphasis is placed on correct reactor design which, as will be shown, reduces to choosing the shape of the reactor, the method of aeration and the type, size, density and size distribution of the particles in the bed.

The main points of the review are summarized in the next main section (pages 152–154) and explored in more detail in later sections. Two clarifications of nomenclature are needed before beginning.

EXPANDED AND FLUIDIZED BEDS

The term ‘expanded bed’ appears frequently in the literature to describe a bed that is expanded only slightly above its settled (packed-bed) height, the adjective ‘fluidized’ being reserved for taller beds. There is no basis for this distinction in the phenomena of fluidization, so ‘fluidized-bed’ refers here to all beds in which the particles are not in continuous contact with each other due to the flow of a fluid up through them.

SPECIFYING REACTOR PERFORMANCE

Three quantities will be used to define the design and performance of the reactors: they are the superficial liquid velocity, which is the volumetric liquid flow rate per unit cross-sectional area of the bed, the substrate conversion, which is the fraction of the substrate fed into the bed that is consumed, and the volumetric productivity, which is the mass of product produced (or substrate consumed) per unit volume of reactor per hour. Note that the substrate cost per unit mass of product is inversely proportional to the substrate conversion and the reactor cost (assumed, proportional to its volume), is inversely proportional to the volumetric productivity. In the wastewater treatment literature results are usually given in terms of fractional removal, synonymous with substrate conversion, and loading, defined as the mass of substrate fed per hour per unit volume of reactor. The volumetric productivity (kg of substrate removed/m³.h) is then equal to (loading × fractional removal).

Characteristics and applications of fluidized-bed bioreactors

IMPORTANCE OF PARTICLE SETTLING VELOCITY

Fluidized-bed bioreactors are necessarily immobilized-cell reactors. Immobilized-cell particles are retained in the bed by gravity while individual cells are so small and light that they are washed out. The critical parameter here is the settling velocity. Particles with a settling velocity less than the superficial liquid velocity are washed out, while those with a larger settling velocity are retained in the bed. It is this retention that allows fluidized-bed fermentors to be run at dilution rates much higher than the cell growth rate.
PARTICULATE VS AGGREGATIVE FLUIDIZATION

Most combinations of liquids and solid particles display what Leva (1959) calls particulate fluidization. The bed expands smoothly with none of the violent bubbling and particle motion characteristic of gas/solid fluidization, known as aggregative fluidization. With little tendency for the solids to move around and mix, a bed containing particles with a range of settling velocities tends to stratify with large heavy particles at the bottom and small light ones at the top. This stratification is reduced but not eliminated by the mixing created by the gas bubbles that are present in many processes either as a nutrient (air) or a product (CO₂, CH₄).

NUTRIENT AND METABOLITE TRANSFER

The disadvantage of immobilized-cell systems is that nutrients can only reach cells inside the particle, and products can only get out, by molecular diffusion which is a very slow process. This intra-particle mass transfer restriction, analysed in the section on Particle Size (pages 160–165), controls the size of particle that should be used. It is particularly serious for aerobic fermentations because the low solubility of oxygen in the media means that it is rapidly exhausted inside the particle.

SURFACE SHEAR

Compared with stirred-tank immobilized-cell reactors, the shear at the particle surface is much lower in a fluidized bed. This is a definite advantage with delicate cells such as those involved in tissue culture. However, it may be a problem with bacteria and yeast because they can continue to grow as a biofilm around the particle. Increasing biofilm volume forces the bed to expand and the steady state, where cell growth equals the rate that cells are washed off by the surface shear, may happen only with an undesirably tall bed. Particles must then be removed from the bed and washed to remove the excess biomass.

LIQUID MIXING

Another advantage of the fluidized bed over the stirred tank is that the flow of liquid through it approximates plug flow. This maximizes the volumetric productivity for any process in which the metabolic rate is a decreasing function of substrate conversion as it is, for example, when an inhibitory product is formed. For zero-order kinetics where the rate is independent of conversion as, for example, in the treatment of concentrated wastewaters, mixing of the liquid does not affect the productivity, while for substrate inhibition (negative order kinetics) mixing is an advantage. It can be introduced into a fluidized-bed system by means of liquid recycle, a recycle ratio above 10 producing in effect a stirred-tank reactor.

Plug flow can also be achieved in packed beds, but a packed bed of the small particles dictated by the intra-particle mass transfer considerations would in
most circumstances be clogged by growth of biomass or by gas bubbles. With the large particles that must be used in practice to avoid clogging, packed-bed reactors typically have productivities an order of magnitude lower than well-designed fluidized beds.

**BATCH VS CONTINUOUS REACTORS**

Fluidized beds are inherently continuous reactors. They therefore share the problems of contamination, back mutation and genetic instability common to all continuous fermentors containing weakened, mutated or genetically engineered cell lines. Together with the oxygen transfer problem, this explains why fluidized beds have not been adopted for the classic aerobic fermentations (antibiotics, amino acids, etc.). They have been most successful in large-scale processes that use stable cell lines, including wastewater treatment and bulk anaerobic fermentations (ethanol, lactic acid, etc.). Among aerobic processes they are used for tissue culture where the fragility of the cells makes the low shear a definite advantage, and the low metabolic rate makes slow oxygen diffusion less of a limitation. They are also used for aerobic wastewater treatment where the solubility of oxygen, although low, is only one order of magnitude less than the concentration of organic matter, not the two or three typical of aerobic fermentations designed to maximize production of metabolites.

**PARTICLE DENSITY**

An important point in the design of fluidized-bed bioreactors is often overlooked. This is the dominant influence of particle density. With the particle size fixed by mass transfer limitations, the size distribution fixed by the need to encourage (or discourage) bed stratification, and the desirability of liquid recycle fixed by the inherent kinetics of the process, the density is the only parameter that the reactor designer is free to choose. Once the density is chosen, the superficial velocity is fixed by the need to fluidize the particle, which fixes not only the shear at the particle surface but also the height of the bed required to provide the needed hydraulic residence time. High-density particles mean high liquid velocity, high shear, and beds with high aspect (height/diameter) ratios, and vice versa. This is in marked contrast to packed beds where the liquid velocity can be chosen, within reason, independently of the size and density of the particles.

**Im mobilized cell particles**

**FLOCS, PELLETS AND GRANULES**

Most types of immobilized cell particles have been used in fluidized-bed bioreactors. The simplest are particles of pure biomass, flocs, pellets, or granules formed naturally by certain strains of micro-organisms.
Figure 1. Types of fluidized-bed bioreactors: (a) tower fermentor; (b) upflow anaerobic sludge blanket reactor; (c) supported film fermentor; (d) aerobic recycle reactor; (e) three-phase aerobic reactor; (f) tapered bed.
In the 1960s several large-scale (3 m × 30 m high) reactors were built for the continuous brewing of beer. They were known as tower fermentors (Figure 1a) and consisted of fluidized beds of a specially chosen strain of *Saccharomyces cerevisiae* (Greenshields and Smith, 1971). Although they were technically successful, being operated continuously for periods of several months with few problems of clogging or contamination, some detailed difference between continuous and batch brewing apparently affected the taste of the beer so the process was discontinued. It may be revived for the production of fuel ethanol from corn, sugar-cane, etc. Laboratory-scale reactors containing flocculant *S. cerevisiae* have shown productivities of the order 10 g/l/h with virtually complete conversion of a 15% glucose feed (Prince and Barford, 1982; Jones et al., 1984). This is far superior to the 1 g/l/h expected from batch reactors. The flocculant strain of the bacterium *Zymomonas mobilis* appears even more productive. This bacterium grows faster than yeast, produces more cohesive and uniform flocs 2–3 mm in diameter, and gives a slightly higher ethanol yield as it uses the less efficient Entner–Doudoroff catobolic pathway. Strandburg, Donaldson and Arcuri (1982) and Scott (1983) have found productivities of 50 g/l/h in laboratory-scale fluidized beds of this organism (even higher values have been claimed, but only for substrate conversions that would be uneconomical in practice).

Some of these reactors have been run for several weeks with unsterilized feed and no problems of contamination by foreign organisms. This is partly due to conditions in the ethanol fermentation which are such that few contaminant organisms can grow—Andrews and Fonta (1986) experienced more difficulties when they switched to the lactic acid fermentation. However, it also reflects the fact that foreign organisms will be washed out of the reactor unless they can attach themselves to the particles or the walls of the bed.

The application of these reactors to aerobic processes has been less successful as a consequence of limitations on the diffusion of oxygen into large flocs and pellets. Smith and Greenshields (1974) give some results on the aerobic growth of yeast and *Aspergillus niger* pellets and the production of vinegar and citric acid. There are, however, a number of important aerobic bioreactors in which small flocs are suspended in the liquid by bubble motion and may be retained in the reactor by gravity. These are properly classified as airlift or bubble-column fermentors rather than fluidized bed reactors unless the particles are deliberately fluidized by an upflow of liquid. However, this distinction is blurred when the rising bubbles induce liquid flow, as in the airlift loop fermentor (Taylor and Senior, 1978).

Beds of biomass particles, known as Upflow Anaerobic Sludge Blanket (UASB) reactors (Figure 1b), have also been developed for anaerobic wastewater treatment. They have been successfully applied on a large scale to the treatment of food processing wastes, landfill leachates, and raw domestic sewage (Lettinga et al., 1984; Fernandes, Cantwell and Mosey, 1985). Reactor productivities vary from 1 gram of chemical oxygen demand (COD) per litre per hour for slaughterhouse waste to 0.03 g COD/l/h for the more dilute domestic sewage (Lettinga et al., 1983). A similar concept has been proposed for the denitrification of wastewater (Klapwijk, Smit and Morre, 1981).
Talking of 'flocs' is inappropriate in these reactors because the light, variable flocs formed by anaerobic digestion organisms are undesirable. They wash out of the reactor too easily, particularly when gas bubbles formed in the process stick to them (Lettinga et al., 1982). Under the right conditions the biomass forms into more regular and denser 'granules' in the millimetre size range. Details of the granulation process are not well understood but it is known to be affected by the type of methanogenic bacteria present, the concentration of bivalent cations, the liquid velocity and the inlet substrate concentration (Wiegant and de Man, 1986). The presence of inorganic particles, either natural sulphide precipitates or added brick dust (Fernandes, Cantwell and Mosey, 1985) apparently helps the process by providing nuclei around which granules can form.

SUPPORTED FILM PARTICLES

In the design and operation of fluidized-bed bioreactors, a major disadvantage of natural flocs and pellets is that neither their size nor their density are under the control of the designer. Some organisms, like the flocculant strain of Z. mobilis, just happen to produce particles of a convenient size, cohesiveness and density for use in a fluidized bed. Others do not and, until the physiological basis of microbial flocculation is better understood, it will not be possible to produce mutant strains with the desired characteristics. Furthermore the design of fluidized beds often calls for particles much denser than natural flocs. One solution to these problems is to grow a film of organisms on a solid support particle. Many different particles have been used including sand, gravel, coal, ion-exchange resin, PVC, ground glass and garnet, with particle diameters ranging from 50 to 1000 µm. Andrews (1986) has discussed the problem of finding the optimum particle size and density for a given application.

Bland et al. (1982) demonstrated a reactor containing vermiculite particles coated with a film of Z. mobilis for ethanol production. Most other applications have used sand or coal particles for wastewater treatment including nitrification (Williams, 1986), and denitrification (Francis and Hancher, 1981; Kurt, Dunne and Bourne, 1987), as well as the removal of organic matter. In fact this is the most-used and best-studied type of fluidized-bed bioreactor. Theoretical and experimental work has gone beyond the normal examination of steady-state reactor performance to consider the dynamic behaviour, which is important because of the variable flow rates and compositions typical of wastewaters. Worden and Donaldson (1987) give a detailed study of the effect of transient changes in concentration on an aerobic reactor treating a phenolic wastewater. Encena and Fernandes (1987) give some experimental results for anaerobic beds.

Fluidized beds of supported biofilms used to remove organic matter from wastewater under anaerobic conditions have been given a confusing array of acronyms including AFB (anaerobic fluidized bed), AAFEB (anaerobic attached film expanded bed) and IFCR (immobilized fluidized cell reactor). They are all essentially identical. Their development can be traced through the work of Hickey and Owens (1981), Jewell, Switzenbaum and Morris (1981),
Henze and Harremoes (1983) and Chen, Li and Shieh (1985). Toldra et al. (1986) give some results from a bed carrying out only the methanogenesis step of the anaerobic process. One difficulty with these reactors is that the start-up period is long because the cells grow so slowly. Stronach et al. (1987) have studied how start up may be accelerated.

Productivities of these reactors are typically 1 gCOD/l/h, similar to UASB reactors and an order of magnitude higher than for the packed reactors known as anaerobic filters (Henze and Harremoes, 1983). Most use considerable liquid recycle (Figure 1c). This is partly because the heavy 20–30 mesh sand support particles commonly used require recycle if the bed is not to be impractically tall and thin (see Section on the Mechanics of fluidized beds, pages 166–170) and partly because the liquid mixing created by recycle makes the reactor easier to control and better able to absorb shock loads of organic or inhibitory material. Variable recycle also allows the constant flow rate through the bed, required to maintain fluidization, to be matched with the variable flow rate of the wastewater.

For aerobic processes (Cooper and Atkinson, 1981; Jeris, Owens and Hickey, 1977) liquid recycle also provides a stream that can be aerated rapidly enough to provide sufficient oxygen for the process (Figure 1d). Hogrefe, Grossenbacher and Hulter (1986) obtained good results using this type of reactor for the removal of S-triazines, compounds relatively resistant to biodegradation. The alternative approach to aeration, bubbling air directly through the bed (Figure 1e), has often been avoided in the belief that the gas bubbles may strip biofilm from the particles. In fact this stripping action is never complete, and it can be an advantage since biofilms in these aerobic reactors need to be kept thin to prevent oxygen limitation within the particles (see Section on Particle size, pages 160–165).

The most extensive studies of three-phase aerobic beds have been carried out by Fan and his co-workers (Fan et al., 1987). After examining various possibilities they adopted a design in which air is bubbled through a draught tube in the centre of the bed, creating some recirculation of both solids and liquid around the bed. This design was found to give the highest oxygen transfer rate. It is on the borderline between a three-phase fluidized bed and an airlift immobilized-cell reactor. The solids hold-up was relatively low (approximately 10%) and the strong agitation generated by the bubbles kept the film thickness in the desirable range of 30–50 µm, at least until the appearance of a filamentous organism caused a rapid film expansion reminiscent of ‘bulking’ activated sludge. Productivities of the order of 1 g/l/h were obtained for the removal of phenol using supports 307 µm in diameter.

POROUS SUPPORTS

There are two types of porous support particles. In macro-porous supports the pores are large and the cells grow inside them. Plastic and stainless steel mesh and porous glass are typical materials (Anonymous, 1987; Atkinson, Black and Pinches, 1981; Black et al., 1984). In the system designed by the Verax Corporation for the production of monoclonal antibodies (Dean et al., 1986;
Young and Dean, 1987) hybridoma cells are grown inside small highly porous beads of collagen. Aeration is via a hollow-fibre cartridge in the liquid recycle stream (Figure 1d).

Jones et al. (1986) immobilized fungi in porous Celite* particles and used a three-phase fluidized bed (Figure 1e without the draught tube) to produce antibiotics continuously. The shear in this bed was sufficient to prevent the fungi from growing on the outside of the particles although Kim et al. (1986) reported significant biofilm accumulation in a very similar system. The difficulties of intra-particle oxygen transfer in this type of system are stressed in this review. However, Jones et al. (1986) make the excellent point that for mycelial fermentations these difficulties must be compared with the oxygen transfer difficulties arising from the presence of the mycelia in a conventional fermentation broth. The oxygen transfer path bubble → cell-free media → particle centre may indeed be easier than bubble → media plus mycelium.

In micro-porous supports such as activated carbon the cells are too large to enter the pores but grow as a film on the outside. Absorption of organic molecules in the pores can both reduce the concentration of inhibitors to which the cells are exposed and store substrate for later metabolism (so-called ‘bioregeneration’ of the carbon; Andrews and Tien, 1981). These advantages have been incorporated into fluidized-bed reactors for wastewater treatment by Weber, Friedman and Bloom (1973). They have been shown to be particularly effective for phenolic-type wastewaters because organic compounds of this category are inhibitory to micro-organisms at high concentrations and adsorb well on activated carbon (Harper et al., 1983; Fan et al., 1987). Fox, Suidan and Pfeffer (1988) found excellent bacterial activity in an anaerobic fluidized bed of activated carbon treating a synthetic coal-conversion wastewater, but none in a packed bed of berl saddles treating the same waste.

During steady-state operation the activated carbon eventually becomes saturated with organics and should then behave like a non-adsorbing support particle. However, true steady-state operation is rare in wastewater treatment and the adsorptive capacity of the carbon continues to provide a buffer against shock loads of inhibitory compounds. The adsorptive equilibrium will also be upset if the particle moves around the bed between regions of low and high concentrations. This may happen randomly due to solids mixing (Andrews and Tien, 1982). In the reactor described by Andrews and Fonta (1986) for the production of ethanol and lactic acid it is done deliberately. Particles fed into the base of the bed adsorb substrate, thus reducing substrate inhibition. As the particles move up the bed the substrate desorbs and is replaced by product, thus reducing product inhibition. Particles removed from the top of the bed are treated to remove product and recycled.

GEL-IMMOBILIZED CELLS

Cells immobilized in gel spheres are normally incorporated in upflow packed-bed reactors. However in the ethanol fermentation the large amount of

* Celite is a registered trade mark of the Manville Corporation.
gas produced and the possibility of cells ‘leaking’ and growing as a film outside the particles can lead to short circuiting of a packed bed, problems that can be avoided by fluidizing the particles (Cho and Choi, 1981; Margaritis and Wallace, 1982; Davison and Scott, 1986). Shirai et al. (1987) applied the same ideas to the production of monoclonal antibodies. The gentle environment in a fluidized bed was crucial in this case because the gel was weakened by the tendency of the calcium that solidifies the alginate to react with phosphate in the media. The particle compression experienced in a packed bed or the shear in an agitated tank would have destroyed the soft, partially decalcified particles.

SOLID SUBSTRATES

For solid substrate fermentations, including the bioprocessing of coal and ores, a fluidized bed of substrate with attached cells must always be considered as a possible reactor configuration. Kleijntjens, de Boks and Luyben (1986) have demonstrated a reactor in which cellulose fibres are fermented directly to ethanol by attached Clostridium thermocellum. The process inevitably reduces the settling velocity of the fibres which then tend to wash out of the bed, so that separating the partially processed fibres (and the cells they contain) from the product and recycling them to the bed is an important problem.

Some solid-substrate fermentations are based on the idea of keeping the liquid volume to the absolute minimum required for the micro-organisms. Tanaka, Kawaide and Matsuno (1986) have shown how this can be achieved in a bed of substrate fluidized by air.

Particle size

STATEMENT OF THE PROBLEM

The appropriate diameter for an immobilized-cell particle is restricted by the ability of essential nutrients to diffuse in and of the products to diffuse out. If it is made too large there will be a zone in its centre in which the micro-organisms are inactive because either the concentration of a limiting nutrient has reached zero (Figure 2b) or the concentration of a product has reached inhibitory levels. This inactive zone represents wasted space in a bioreactor and therefore reduces its productivity.

Consider a spherical particle which may be a biofilm growing on a solid support of radius $R$ (Figure 2) or a floc, pellet, gel sphere or porous support in which case $R = 0$. The cell concentration in the particle, $X$, is assumed to be uniform (Dahli and Chau, 1987, discuss this point for gel spheres), and the mass-transfer resistance in the liquid phase around the particle is ignored (Andrews and Przedziecki, 1986 show this is valid in fluidized beds, except for some aerobic wastewater treatment situations). Using the nomenclature listed on page 173, a mass balance over an element $dr$ then gives:
Figure 2. Concentration profiles in a biofilm. (a) Thin film; (b) thick film.

\[
\frac{D_j}{r^2} \frac{dr}{dr} \left( r^2 \frac{dC_j}{dr} \right) = q_j X
\]

\[C_j = S_j \text{ at } r = R + L\]

\[\frac{dC_j}{dr} = 0 \text{ at } r = R\]  \hspace{1cm} (1)

There is one such equation for every nutrient diffusing into the particle and one for every product diffusing out. For products the right-hand side of the equation is negative, but this can be avoided by the variable transformation \( C_p = \tilde{p} - (\text{actual product concentration}) \) where \( \tilde{p} = \text{completely inhibitory product concentration} \). Note that the condition \( C_p = 0 \) now stops metabolism just as it does for a nutrient. Solving these equations gives the concentration profiles shown in Figure 2. However, the solution is difficult because the consumption (or production) rate of a component \( q_j \) (a ‘component’ is a nutrient or product) is, in general, a function of all the component concentrations at a point. Two approaches have been used. In the first (Shieh, Mulcahy and LaMotta, 1981) these complex kinetic expressions are approximated by simple zero-order or first-order kinetics and the equations solved analytically. In the second the
complete kinetic equations (even structured models) are used and the
equations solved numerically. The work of Droste and Kennedy (1986) on
anaerobic wastewater treatment and Park, Davis and Wallis (1984) on
penicillin production are examples of this approach.

The analysis given here, based on the work of Andrews (1988), represents a
compromise between the mathematical simplicity of the first approach and the
accuracy of the second. The first step is to identify the two components that
have the most influence on the metabolic rate. Writing the specific
consumption rate as \( q_i = \mu Y_i \) allows the specific growth rate, \( \mu \), to be
eliminated between all the equations (1). The resulting equations can be
integrated to give a relationship between all the component concentrations in
the particle:

\[
D_1 Y_1 (S_1 - C_1) = D_2 Y_2 (S_2 - C_2) = \ldots D_m Y_m (S_m - C_m)
\]  

(2)

It follows that the limiting component, the concentration of which will
reach zero first in a large particle (Figure 2b), is the one with the lowest value of
the product \( DYS \). Note that this is not necessarily the same as the limiting
component for the reactor, which is the one with the lowest value of \( K \)\( Y_s \),
where \( S_i = \) inlet concentration. The rate-controlling component is defined as
the one, the concentration of which in the biomass first reaches the value which
restricts \( \mu \) to 90% of its value under liquid-phase conditions. For Monod
kinetics this makes the rate-controlling component the one with the lowest value of

\[
[DYS(\frac{S + K}{S + 10K})]_j
\]

This is the same as the limiting component if \( S >> K \) for all components, but
otherwise they may be different.

The kinetic equation for the consumption of the rate-controlling component
(no subscript) can now be written:

\[
q = \frac{\mu}{Y} = \frac{\tilde{\rho} C}{K + C} H(C_i)
\]  

(3)

\( H(C_i) \) is the Heavyside function which equals 1 when \( C_i > 0 \) and 0 when \( C_i = 0 \). When the limiting and rate-controlling nutrients are the same it is redundant.

When an inhibitory product is being formed it is quite common for it to be the
rate-controlling component while the substrate is the limiting component.
Equation (3) can also approximate this situation because with \( C_p \) defined by the
variable transformation given previously and \( K \) replaced by \( \tilde{\rho} \), the low \( C \)
asymptote of Equation (3) corresponds to the linear inhibition model.

EFFECTIVENESS FACTORS

Once the important components have been identified, Equation (3) can be
substituted into the Equation (1) for the rate-controlling component. The
general solution of the resulting differential equation is not known, so
asymptotic solutions are derived for small $S$, large $S$, large $(R + L)$ etc. These solutions are used to evaluate an effectiveness factor defined by:

$$
\eta = \frac{\text{actual consumption (production) rate of component in particle}}{\text{consumption (production) rate if whole particle contained biomass exposed to liquid-phase conditions}}
= \frac{3D}{(R + L)Xq(S)} \frac{dC}{dr} \bigg|_{(R + L)}
$$

(4)

The effectiveness factor is a dimensionless metabolic rate. For the kinetics of Equation (3) the correct definition of dimensionless distances in the problem is:

$$
\theta = \frac{LS}{K + S} \left\{ \frac{qX}{2DK\left[\frac{S}{K}(1 - E) - \ln\left(\frac{K + S}{K + ES}\right)\right]} \right\}^{1/2}
$$

(5)

$$
E = 1 - \frac{(DYS)}{(DYS)}
$$

Replacing $L$ by $R$ gives a similar dimensionless support particle radius, $\gamma$. The parameter $E$ gives the relative importance of the limiting and rate-controlling components in fixing the rate. It falls between zero when one component is both limiting and rate controlling, and 0.9, a consequence of the definition of the rate-controlling component.

**Figure 3.** Particle effectiveness; $-S<<K$ and $E = 0$; $-S>>K$ or $E = 1$. 
The solution for the effectiveness factor as a function of the dimensionless particle radius ($\theta + \gamma$) is shown in Figure 3. The significant feature of this graph is that, with the definitions of $\theta$ and $\gamma$ used here, the two asymptotic solutions are sufficiently close together to allow some useful generalizations.

RESULTS

With the definition given by Equation (4) the reactor volumetric productivity equals the value of $\eta e X_q(S)$ averaged through the reactor. Thus a high productivity requires a high average value for the effectiveness factor. Figure 3 shows that $\eta$ is highest for particles which have biomass throughout ($\gamma = 0$) and a radius $\theta < 1$. Estimates of what $\theta = 1$ means in millimetres are given in Table 1. They are based on the assumption that diffusivities in flocculated biomass and synthetic gels are one-half of the values in water. This is only approximately true (Ngain and Lin, 1977; Hannoun and Stephanopoulos, 1986) but the diffusivity is not a critical parameter as only its square root appears in the calculation.

The solubility of oxygen in water is so low that oxygen is almost certain to be both the limiting and rate-controlling component whenever the concentration of organic matter to be metabolized is above 100 g/m$^3$. For oxygen diffusion from air-saturated water into a packed mass of cells, $\theta = 1$ corresponds to a radius of less than 100 $\mu$m. This is a typical size for activated sludge flocs, but is too small to be dealt with conveniently in a fluidized bed (see Section on the mechanics of fluidized beds, pages 166–170). The figure would be the same for cells packed in a porous support, because both $X$ and $D$ would then be multiplied by the internal porosity of the particle. For tissue culture cells the metabolic rate is an order of magnitude lower, which increases the desirable radius to a more practical 200 $\mu$m, close to the size used by Young and Dean (1987).

In anaerobic processes oxygen diffusion is not a problem and the particles can be much larger. The figures for anaerobic wastewater treatment in Table 1 show that, as expected, the allowable particle size increases with the concentration of the limiting component. A major advantage of the fluidized bed is that this situation is created automatically by the tendency of the bed to stratify with the largest particles near the reactor entrance where the nutrient concentration is highest. The same applies to the ethanol fermentation (analysed based on the linear inhibition model). If the design requirements of a reactor call for 99% conversion of a 150 g/l glucose solution, then a bed of gel-immobilized cell particles with radii in the range 0.25–1.6 mm would stratify correctly to give high effectiveness factors throughout the reactor.

The curves in Figure 3 for biofilms on solid support particles ($\gamma > 0$) show that there is a definite optimum film thickness. Consider for example the case of $\gamma = 0.5$. Adding a monolayer of cells, which gives $\theta$ of order 0.01, gives a very low effectiveness and therefore a low reactor productivity. Adding more cells up to $\theta$ approximately 1 greatly improves the effectiveness, but any further increases in film thickness are counterproductive due to the mass transfer limitations in the film. Note that it is only possible to reach a high effectiveness factor with
**Table 1.** Estimates of optimum particles radius \((d = 1)\)

<table>
<thead>
<tr>
<th>System</th>
<th>Limiting component</th>
<th>Rate-controlling component</th>
<th>Cell concentration (g/ml)</th>
<th>(\dot{q} ) [g/l]</th>
<th>(\frac{DK}{\dot{q}X}) [mm]</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aerobic wastewater treatment</td>
<td>(O_2)</td>
<td>(O_2)</td>
<td>70</td>
<td>0.2</td>
<td>0.7</td>
<td>Hoehn and Ray (1973)</td>
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<tr>
<td></td>
<td>(S_o = 8 \text{ g/m}^3)</td>
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<tr>
<td></td>
<td>(&gt;&gt;K_o)</td>
<td></td>
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<td></td>
<td></td>
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<tr>
<td>Tissue culture in porous supports</td>
<td>(O_2)</td>
<td>(O_2)</td>
<td>80</td>
<td>0.03</td>
<td>0.17</td>
<td>Dean <em>et al.</em> (1986)</td>
</tr>
<tr>
<td></td>
<td>(S_o = 8 \text{ g/m}^3)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(&gt;&gt;K_o)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Anaerobic wastewater treatment</td>
<td>COD</td>
<td>COD</td>
<td>(\dot{q}X = 3 \text{ g/l h})</td>
<td></td>
<td></td>
<td>Henze and Harremoes (1982)</td>
</tr>
<tr>
<td></td>
<td>(K = 200 \text{ g/m}^3)</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td>(S = 100 \text{ g/m}^3)</td>
<td></td>
<td></td>
<td></td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(S = 1000 \text{ g/m}^3)</td>
<td></td>
<td></td>
<td></td>
<td>1.2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(S = 10000 \text{ g/m}^3)</td>
<td></td>
<td></td>
<td></td>
<td>3.0</td>
<td></td>
</tr>
<tr>
<td>Ethanol from gel-immobilized <em>Z. mobilis</em></td>
<td>Glucose</td>
<td>Ethanol</td>
<td>10</td>
<td>5.1</td>
<td>1.6</td>
<td>Andrews (1986); Scott (1983)</td>
</tr>
<tr>
<td>Inlet conditions</td>
<td>(S_o = 150 \text{ g/l}); (\rho = 0)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Glucose</td>
<td>Glucose</td>
<td>0.25</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Outlet conditions</td>
<td>(S_o = 1.5 \text{ g/l}); (\rho = 71 \text{ g/l})</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
small support particles. With $\gamma = 0.5$ (radius equal to one-half of the values shown in Table 1) the maximum effectiveness is approximately 0.9. The 2 inch ($\approx 51$ mm) rocks typically used in trickling filters correspond to a $\gamma$ of several hundred and thus a maximum effectiveness factor of order 0.01. This illustrates the scale of the possible improvement in productivity obtainable by using smaller supports, and fluidizing the bed to prevent clogging by the biofilm.

**Mechanics of fluidized beds**

**SOLID AND LIQUID HOLD-UP**

Most fluidized-bed fermentors contain gas bubbles either as a nutrient (air) or a product (CO$_2$, CH$_4$, N$_2$). The mechanics of three-phase fluidized beds was recently reviewed by Muroyama and Fan (1985). Unfortunately there is no information on the situation where gas is a product and the gas flow rate increases up the bed. Another difficulty is that wall effects and gas slugging appear in these beds unless the bed diameter is very large compared with the sizes of the particles and the gas bubbles. Caution is needed in applying results from laboratory columns to full-scale beds.

When there is no gas and no wall effects the liquid hold-up (porosity) of a fluidized bed is usually given by a correlation of the form introduced by Richardson and Zaki (1954):

$$e^n = \frac{U}{U_t}$$  

(6)

Many correlations have been suggested for the constant $n$. Al-Djibouni and Garside (1979) gave the simplest:

$$\frac{5.1 - n}{n - 2.7} = \frac{Re_t^{0.9}}{10}$$  

(7)

Note that for large dense particles $Re_t$ is large and $n = 2.7$. For small light particles $Re_t$ is small and $n = 5.1$. There is no reason to believe that these correlations would not apply to biological particles (Ngain and Martin, 1980) although some controversy has arisen, probably due to the effect of a coating of biomass on the particle settling velocity (Hermanowicz and Ganczarczyk, 1983). The best correlations are those given by Mulcahy and Shieh (1987).

For thick biofilms growing on solid support particles, Andrews (1982) showed that a settling-velocity correlation of the form $C_D = aRe_t^{-\varepsilon}$ leads to the following equation for the effect of biofilm on settling velocity

$$\frac{U_t}{U_{ts}} = \left(1 + Bx\right)^{1/(2-\varepsilon)} \left(1 + x\right)^{1/3}$$  

(8)

This is plotted in Figure 4 versus the quantity $(1 + x)$, the total particle volume divided by the support particle volume. The important result is that for high density supports (small $B$) the main effect of the biofilm is in reducing the average particle density, so film growth reduces the settling velocity. For low
density supports (large $B$) the main effect is on the particle diameter, so film growth increases settling velocity. There is also a category of support particles ($B \sim 0.25$: support density $\sim 1.1$) for which the two effects cancel out, leaving the settling velocity almost unchanged over a wide range of film volumes. Note that the changes in bed height created by the changes in the settling velocity shown in Figure 4 are relatively small compared with the changes required just to accommodate the film volume. It follows that as a first approximation, the fractional increase in bed height as a biofilm develops is proportional to the film volume, $x$ (Andrews and Tien, 1979).

**SOLIDS MIXING AND STRATIFICATION**

There is clear evidence that particles in a liquid fluidized bed show little tendency to move around and ‘mix’ and a strong tendency to form an ordered, stratified arrangement with high-settling-velocity particles at the bottom and small, light particles at the top (Scarlett and Blogg, 1967). Factors which tend to break down the stratification and encourage mixing, most of which are discussed by Al-Djibouni and Garside (1979), are as follows:

1. A narrow distribution of particle settling velocities: in the limit of monosized uniform-density particles there is no basis for stratification;

2. Low aspect ratio; short, wide beds show greater mixing due to lower wall effects and greater effects from the inevitably uneven flow distribution at the bed inlet;

3. Gas bubbles: the excellent study by Gommers et al. (1986) shows that the effect of gas is much stronger in large-diameter beds, the behaviour of which is not influenced by wall effects;

4. Large, heavy particles; mixing increases with the $Re$ values of the particles until, at a value corresponding roughly to lead shot in water, the fluidization becomes aggregative;

5. Bed inclination; deviations of as little as 1/2 degrees from the vertical can cause large-scale recirculation of the bed solids (Leva, 1959);

6. Solid layer inversion; this is a complex phenomenon observed in bi-modal beds (Moritomi, Iwase and Chiba, 1982). Its implications for fermentors have not been worked out.

It was shown in the Section on particle size (pages 162–164) that the reactor productivity is given by the average value of $[\eta_\infty X q(S)]$, and that one advantage of solids stratification is that it arranges the particles so as to maintain a high effectiveness factor, $\eta$, throughout the bed. The corresponding disadvantage is that the low settling velocity of the particles near the top of the bed leads to a high porosity [Equation (6)] and thus a low solids hold-up, $\epsilon_s$, at this point. This is exacerbated when gas is generated in the bed, because the gas hold-up is inevitably high near the top of the bed. It is worst in beds of flocs because the large flocs near the base of the bed are exposed to the highest
substrate concentrations and therefore grow larger, while the small flocs near the top are exposed to very low substrate concentrations and do not grow. In both UASB reactors and tower fermentors the particles near the base of the reactor can grow so large that the bed there is not actually fluidized and can form a solid plug (Jones et al., 1984). Meanwhile the particles at the top of the bed are so small and light that an expanded disengagement zone is required to keep them in the bed [Figures 1(a) and (b); mechanical devices are also sometimes installed to separate the flocs from gas bubbles Lettinga et al., 1982]. Fernandes, Cantwell and Mosey (1985) studied a UASB reactor 2 m high and found an almost uniform decrease in solids concentration (dry weight/unit volume of liquid) from 150 g/l at the bottom to zero at the top. Greenshields and Smith (1971) found a drop from 300 g/l to 7.5 g/l in the yeast concentration in a tower fermentor.

These measured reductions in particle concentration are the best available evidence that large-scale fluidized beds containing gas bubbles do in fact stratify. Royston (1966) reported a stranger result, a maximum in the solids concentration mid-way up a tower fermentor producing beer. This can be explained by the interaction between particle stratification and the reduction in liquid density and (to a lesser extent) viscosity as sugars are fermented to ethanol. Flocs near the base of the bed may be large but their settling velocity is small because their density is only slightly larger than that of the feed solution. Flocs near the middle of the bed may be smaller but their settling velocity, and thus the solids hold-up, is higher because the liquid density has dropped. For a floc of density 1.03 g/cm³, the drop in liquid density from a 10% glucose solution (1.02 g/cm³) to a 5% ethanol solution (0.99 g/cm³) would quadruple \( U_i \). For much denser particles the effect would become negligible.

The stable stratification described above for a bed of flocs can never happen with a bed of dense support particles coated with biofilm, because in this case the growth of biomass decreases the settling velocity (Figure 4). The biomass on a particle near the base of the bed where substrate concentrations are highest still grows fastest, but this growth now reduces \( U_i \) which tends to move the particle up the bed. Evidence of stratification based on biofilm thickness in large-scale reactors is not clear (Shieh, Sutton and Kos, 1981) partly because the variations in \( U_i \) caused by film growth (Figure 4) are less than the variation in \( U_{is} \) among the support particles. This type of stratification has been observed in a laboratory-scale bed of monosized activated carbon (Andrews and Fonta, 1986).

The best solution to the problem of decreasing effective cell concentration up a fluidized bioreactor is to make \( U \) a variable by tapering the bed (Figure 1f). This has been done by Scott (1983) and Boening and Larsen (1982) in the laboratory, but there are some difficulties in constructing a large-scale reactor. The taper angle is a critical parameter because if it is too large the bed tends to 'spout', with a jet of fluid moving rapidly through a region of low solids concentration in the centre of the bed, and a concentrated bed of solids moving slowly downwards near the wall. Spouting implies some short-circuiting of the bed and would normally be considered undesirable, but Webb, Fukuda and Atkinson (1986) discuss some advantages of this arrangement for cellulase production.
LIQUID MIXING

The exact liquid-mixing condition in single-pass reactors is not well understood. In the absence of gas bubbles, theoretical considerations suggest that it would be close to plug flow (Andrews, 1982). Gas increases mixing (Muroyama and Fan, 1985; Gomers et al., 1986). The UASB reactor probably has the best-mixed liquid due to its gas production and low aspect ratio. Bolle et al. (1986) found that their tracer data for a full-scale UASB fit a reactor model consisting of two stirred tanks with short-circuiting caused by the gasflow.

What is certain is that mixing is not intense enough to destroy completely the gradients of concentration in the liquid phase through the bed. This makes solids mixing highly undesirable, because an individual particle would then move rapidly between regions of high and low concentrations. The cells would be unable to acclimatize themselves and would therefore not work efficiently. This is particularly important for sequential fermentations such as anaerobic digestion, where not only the concentration but also the composition of the substrates can change. A particle containing mainly methanogenic bacteria would not function well if random motion put it near the base of the bed where the substrates are mainly complex organic molecules rather than the acids required by methanogens. The advantages of encouraging bed stratification in these situations have been discussed by Yang (1987).

The plug flow assumption is a good approximation for some reactors and gives an upper limit against which the performance of real reactors can be measured. The mass balance equation for any component is:

$$-U \frac{dS}{dZ} = \epsilon, \eta qX$$

$$S = S_i \text{ at } Z = 0$$
This cannot be solved unless we know how the solids holdup \( \varepsilon_s \) varies with distance up the bed \( Z \). This can be avoided by changing the independent variable to \( \nu \), the volume of solids per unit area below a point in the bed. It is related to \( Z \) by:

\[
\frac{d\nu}{dZ} = \varepsilon_s
\]  

(10)

Suppose that the particle size can be controlled to keep \( \eta \) constant and \( q \) is given by the Monod equation [Equation (3)]. Dividing (9) by (10) and integrating from the inlet to the outlet gives the total solids volume required per unit area of reactor:

\[
\nu = \frac{U}{\eta q X} \left[ S_i y - K \ln(1 - y) \right]
\]  

(11)

\[
y = 1 - \frac{S_{\text{out}}}{S_i} = \text{substrate conversion.}
\]

Many fluidized-bed bioreactors are deliberately made effectively into continuous stirred tanks by introducing a large amount of liquid recycle [Figure 1(c)(d)]. This has some advantages for ease of aeration, reactor control and the ability to absorb the slug loads of inhibitory material often found in wastewaters. However, it usually involves some penalty in reactor productivity.

The exceptions are fermentations involving substrate inhibition, when liquid mixing can increase the reaction rate by decreasing the substrate concentration to which the cells are exposed. Klein and Kressdorf (1986) have designed a reactor for producing ethanol using gel-immobilized Zymomonas mobilis, which is known to be substrate inhibited. It consists of two fluidized beds in series each with considerable liquid recycle, followed by a plug-flow packed bed to finish the fermentation. Only a detailed analysis, not currently available, could show whether this complex arrangement is preferable to a single column. It must not be forgotten that the intra-particle mass transfer resistance performs the same function as liquid mixing: it reduces the substrate concentration to which the cells are exposed. It follows that in substrate-inhibited situations the mass transfer resistance will increase the rate, effectiveness factors can be greater than one, and particles can be much larger than in substrate-limited or product-inhibited situations.

**The optimum reactor**

**DESIGN STUDY**

The accumulated knowledge reviewed in the previous sections still does not allow us to specify an optimum type of particle or reactor for a given application. Take, for example, the relationship between reactor productivity and substrate conversion derived from Equation (11):

\[
\text{Productivity} = \frac{\varepsilon_s U S_i y}{V} = \frac{\varepsilon_s \eta q X}{1 - \frac{K S_i}{S_i} \frac{\ln(1 - y)}{y}}
\]  

(12)
Fluidized-bed bioreactors

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Would his be higher with gel-immobilized cells or flocs (assuming that a suitable flocculant strain is available)? The size of gel particles can be closely controlled allowing a high, almost uniform solids hold-up and a high effectiveness factor. The size of flocs can only be crudely controlled (by draining out over-large flocs from the base of the bed, for example) so the effectiveness factor and solids hold-up (see Section on solids mixing and stratification, pages 167–168) will be lower. On the other hand, the cell concentration, \(X\), in a floc may be an order of magnitude higher than in a gel particle, so the overall reactor productivity is not necessarily lower.

Other problems can be illustrated by a design study. Suppose we have \(F = 10\ m^3/h\) of wastewater containing 100 g/m\(^3\) of BOD, and propose to achieve 90% removal in an aerobic, supported biofilm reactor. Pure-oxygen aeration of the recycle stream (Figure 1d) gives a dissolved oxygen concentration of 35 g/m\(^3\) at the fluidized-bed inlet. The oxygen concentration at the outlet is to be kept above 1 g/m\(^3\) to prevent anaerobic activity. Since we know the BOD at the bed outlet is \(S = 10\ g/m^3\) and can assume that the consumption of 1 g BOD requires 1 g O\(_2\) (\(Y/Y_o = 1\)) it follows that the BOD at the bed inlet is \(S_i = 44\ g/m^3\). A mass balance at the mixing point ahead of the aerator shows that this requires a recycle ratio \(RR = 1.6\). The actual flow through the bed is therefore \(F(1 + RR) = 26\ m^3/h\). Reasonable estimates for the other parameters are \(D_o/D = 4\) (based on an average value for organic molecules in water), \(K = 7\ g/m^3\) for BOD and \(K_o = 0.1\ g/m^3\) for O\(_2\). From the equations given in the Section on particle size it follows that the organic matter is both limiting and rate controlling at the inlet, and is also rate controlling at the outlet while oxygen is limiting (\(E = 0.6\)).

We can now go through the following calculation. Pick a dimensionless support particle size, \(\gamma\), find the optimum film thickness, \(\theta\) and the corresponding effectiveness factor, \(\eta\), from Figure 3, and calculate the film volume parameter \(x\) (from geometry \(1 + x = [(L + R)/R]^3 = [1 + \theta/\gamma]^3\)). Convert the chosen \(\gamma\) value to real particle sizes using Equation (4) and the parameter values from Table 1; for example \(\gamma = 1\) corresponds to \(R = 45\ \mu m\) at the outlet and \(R = 74\ \mu m\) at the inlet (100 \times 170 mesh particles). Now for a given particle density calculate the settling velocity of the inlet particle in water, \(U_{is}\), the corresponding settling velocity for a coated particle, \(U_c\) [Equation (8)], and the superficial velocity, \(U\), required to fluidize these particles to \(\varepsilon = 55\%\) [Equations (6), (7)]. Fifty-five per cent is a reasonable value to prevent particle agglomeration. Note that the porosity will increase up to about 65% at the top of the bed due to the effect of stratification, so an average solids hold-up \(\varepsilon_s = 0.4\) will be assumed. The solids volume required in the bed, \(V\), can now be computed from Equation (11), the bed height from \(V/\varepsilon_s\), and the cross-sectional area of the bed from \(F(1 + RR)/U\).

RESULTS

There are clearly a number of approximations in the above calculation, but the results (Figure 5) show a number of important features that remain true whatever approximations are made.

First, the main benefit of this type of calculation is to establish an ideal reactor against which the performance of real reactors can be judged. Any of
the 'ideal reactors' allowed by Figure 5 would be much smaller and more productive than existing fluidized beds used in aerobic wastewater treatment. This is because the existing beds have usually been designed by choosing the size and shape of the reactor, and the size and density of the support particles independently of each other. This inevitably produces a suboptimal design.

Second, this calculation can eliminate many unpromising designs without extended, expensive experimentation. For example, Figure 5 shows that a bed of 50 × 80 mesh stainless steel supports (density ~7.8 g/cm³) would work well, giving a high effectiveness factor and a bed aspect ratio (height/diameter) close to unity. Coal of 25 × 40 mesh (density ~1.5 g/cm³) is also possible, although the productivity would be lower due to the lower effectiveness factor. However, 50 × 80 mesh coal would require a bed 400 cm in diameter by 1 cm high, which does not qualify as a fluidized bed. All smaller particles, whatever their density, and all systems which aerate the wastewater with air instead of oxygen, encounter the same difficulty. Using similar calculations Andrews (1986) showed that a bed of cells immobilized in porous glass particles would need to be impractically tall for ethanol production. Young and Dean (1987) showed the need for the addition of stainless steel to their collagen supports used for tissue culture.

Third, by eliminating the many undesirable particles, results like those in Figure 5 leave only the question of choosing between the possible solutions. This is related to the problem of maintaining in a real bed the optimum film thickness assumed during the calculations. We can calculate how many support particles to put in the bed [V/(1 + x)] but not how to stop the film growing past its optimum thickness. There are two possibilities.

The shear at the biofilm surface depends on the size and density of the support particle. It should therefore be possible to choose a particle such that the steady-state situation, in which the rate of film growth is balanced by the
rate of cell wash-off, endogenous metabolism, etc. at the optimum film thickness. This steady state has been modelled by Andrews (1982), and in a series of papers by Rittman (Rittman and McCarty, 1980; Rittman, 1982; Wang, Suidan and Rittman, 1987). However, no useful values for the film decay parameter are available so the choice of particle must be done empirically. In fact, no systematic experiments along these lines have been reported, and once again a comparison between current practice and our ‘ideal reactors’ shows considerable scope for improvement. Figure 3 shows that $\gamma = \theta = 1$ are excellent choices for support particle radius and biofilm thickness. This corresponds to a film volume ratio $x = 7$, and to accommodate this much biomass the bed must be approximately eight times the height of a fluidized bed of clean support particles at the same superficial velocity (Andrews and Tien, 1979). The presence of gas, which can both increase or decrease bed height (Muroyama and Fan, 1985), complicates the analysis but the order of magnitude of the result is correct. In practice, beds of sand used for the anaerobic treatment of dilute wastewaters hardly expand at all during the start-up period when the biofilm is developing (Toldra et al., 1986; Stronach et al., 1987). The steady-state biofilm is clearly far too thin, and considerable benefits could be expected by changing to a low-density support which generates lower shear at the film surface.

In aerobic wastewater treatment and many fermentations the cells grow faster and produce a more cohesive film, so the steady-state film thickness is usually considerably larger than the optimum. The procedure in this case is to fix the bed height at the required value by removing particles from the top of the bed and stripping off excess biomass in a vibratory sieve (Jeris, Owens and Hickey, 1977) or other device. In this case it is important for the support particles to be virtually monosized, otherwise stratification will leave the larger particles near the bottom of the bed where they will accumulate excessive biofilm without ever going near the washing device. A bed of dense monosized supports naturally tends to stratify based on biofilm thickness with the thickest films near the top. Although this is the wrong way round to maintain the optimum effectiveness factor (Andrews, 1986; $L$ increases as $S$ decreases) it makes the washing device very efficient.

**Nomenclature**

- $a$  Constant in drag coefficient correlation
- $B$  Buoyant density of biofilm/buoyant density of support particle
- $C$  Concentration of a component in bio-particle
- $D$  Diffusivity in bio-particle
- $E$  $1 - (DYS)/DYS$
- $e$  Exponent on $Re$, in drag coefficient correlation
- $F$  Flow rate of wastewater
- $H$  Bed height
- $K$  Monod constant
- $L$  Biofilm thickness
- $n$  Exponent in Richardson–Zaki correlation
\( \hat{p} \)  
Product concentration that completely inhibits cell growth

\( \hat{q} \)  
Maximum specific consumption (or production) rate of a component

\( q \)  
Specific consumption (or production) rate of a component

\( Re_i \)  
Particle Reynolds number based on \( U_i \)

\( r \)  
Radial position

\( RR \)  
Recycle ratio

\( R \)  
Support particle radius

\( S \)  
Concentration in liquid phase

\( U \)  
Superficial liquid velocity

\( U_i \)  
Settling velocity

\( U_{1s} \)  
Settling velocity of a support particle

\( v \)  
Solids volume per unit area below any point in bed

\( V \)  
Solids volume per unit area of bed

\( X \)  
Cell concentration in bioparticle

\( x \)  
Biofilm volume/support particle volume = \((1 + \theta/\gamma)^3 - 1\)

\( y \)  
Fractional substrate conversion

\( Y \)  
Cell yield

\( Z \)  
Distance up the bed

\( \varepsilon_s \)  
Local solids hold-up

\( \bar{\varepsilon}_s \)  
Average solids hold-up

\( \varepsilon \)  
Liquid hold-up (porosity)

\( \gamma \)  
Dimensionless support particle radius

\( \mu \)  
Specific growth rate of cells

\( \theta \)  
Dimensionless biofilm thickness

\( \eta \)  
Effectiveness factor

**SUBSCRIPTS**

\( i \)  
Inlet conditions

\( j \)  
Any component

\( l \)  
Limiting component

\( o \)  
Oxygen

\( P \)  
Product

\( S \)  
Substrate

**References**


FRANCIS, C.W. AND HANCHER, C.W. (1981). Biological denitrification of high nitrate wastes generated by the nuclear industry. In Biological Fluidized Bed Treatment of


